Capital in the Business Cycle: Renting versus Ownership

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Abstract

This paper studies how business cycle dynamics are affected by the possibility to rent as opposed to own productive capital. We provide empirical evidence based on firm-level data for the US which confirms the counter-cyclicality of the share of renting in capital expenditure over the business cycle. We develop a dynamic stochastic general equilibrium model with borrowing constraints and two types of agents, and allow for both owned and rented capital to be used in production. The model focuses on a fundamental trade-off between the two types of capital: only owned capital can serve as collateral, but owning requires more liquidity than renting. The model performs well in replicating the counter-cyclical behavior of the rental share and matching some of the key business cycle moments observed in the data. Finally, simulation results show that the presence of a renting sector alleviates the adverse impacts of a negative financial shock, and these results are confirmed by empirical cross-country evidence.

1 Introduction

How does the ownership of productive capital affect the aggregate behavior of the economy? On the one hand, this question has been somewhat ignored by macroeconomists, mainly because in a neoclassical, frictionless world the question whether firms own or rent productive capital becomes irrelevant. On the other hand, a number of recent finance papers (Eisfeldt and Rampini, 2009; Rampini and Viswanathan, 2010b; Gavazza, 2011) have identified various factors such as financial constraints that make the issue of renting versus owning relevant. The motivation of this paper is to show that the presence of financial constraints makes the question of renting versus owning relevant in understanding the business cycle as well.

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We first report on some empirical evidence based on firm-level data, which shows that the renting of productive capital is counter-cyclical in a sense that the share of renting in total capital expenditure increases in recessionary periods. We then build an aggregate model with two types of agents where both rented and owned capital are explicitly modelled as production inputs. We show that the presence of borrowing constraints can generate a substitution effect between rented and owned capital leading to the counter-cyclicality of renting. At the heart of this substitution effect lies a fundamental trade-off between owning and renting: on the one hand, owning productive capital is advantageous because only owned capital can serve as collateral for borrowing; on the other hand, renting productive capital requires less liquidity which makes renting more attractive in times of financial distress. Our model is related to two major strands of literature.

On the one hand, our paper is related to the recent macroeconomics literature with financial frictions\(^1\). Our theoretical framework is motivated by models with borrowing constraints derived from limited enforcement as in Kiyotaki and Moore (1997), Krishnamurthy (2003) and Matsuyama (2007), as well as by the related growing literature that studies the relationship between limited enforcement and firm’s financing and investment decisions as in Albuquerque and Hopenhayn (2004), Lorenzoni and Walentin (2007), Caggese (2007) and Rampini and Viswanathan (2010b). We draw on the the aggregate implications of firm financing with limited enforcement analyzed by Cooley, Marimon, and Quadrini (2004), which shows that limited enforceability amplifies the impact of aggregate technology shocks leading to higher macroeconomic volatility. In addition, our analysis of the impact of financial shocks is inspired by Jermann and Quadrini (2007) and Jermann and Quadrini (2009) who show how financing constraints can make financial shocks exert large impact on real variables.

On the other hand, our paper is related to the finance literature on capital structure. In the neoclassical investment literature, the ownership of productive capital is irrelevant when markets are assumed to be frictionless. In these models, capital ownership was indeterminate, and firms were assumed to rent all capital.\(^2\) Myers, Dill, and Bautista (1976) and Myers, Dill, and Bautista (1976) are regarded as the first papers to analyze the renting-versus-owning decision in the framework of Modigliani and Miller (1958).\(^3\) The emphasis of the early finance literature was on tax considerations. Our model focuses on other dimensions, such as agency costs associated with the separation of capital ownership and control as analyzed by Smith and Wakeman (1985). They pointed at a major advantage of capital renting, namely, that regaining physical possession of a rented asset is easier than for a secured debtholder to acquire the collateral. As a result, renting preserves capital which may have important implications for firms facing financing constraints. Subsequently, Sharpe and Nguyen (1995) and Eisfeldt and Rampini (2009) explored the link between the easier repossession of rented capital and financing constraints, and they used firm-level data to show that smaller firms, that appear more financially constrained, rent a larger fraction of their capital.

The focus of our paper differs substantially from all those above. We concentrate on the interaction between financing constraints and the cyclicality of rented and owned capital. We show analytically that our

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\(^1\)See Brunnermeier, Eisenbach, and Sannikov (2011) for a recent survey.

\(^2\)It was a standard modelling assumption in the first generation RBC models as well.

\(^3\)The finance literature often refers to renting as leasing.
model with borrowing constraints induces firms to substitute away from owned capital to rented capital in the face of shocks hitting entrepreneurial wealth. This substitution-effect and the associated counter-cyclicality of the rental share in production has important implications to the extent that it mitigates the adverse impacts of financial shocks on the business cycle.

The rest of the paper is organized as follows: Section 2 presents empirical evidence regarding the dynamics of renting of productive capital in the US. Section 3 presents the theoretical model, explains why the rental share is counter-cyclical, and compares the business cycle statistics implied by the model to those found in the data. Section 4 extends the baseline model with asset-prices. Section 5 provides both simulation and empirical results to show that the presence of renting mitigates the adverse impacts of a financial shock on the business cycle. the Section 6 concludes.

2 Empirical evidence

The aim of this section is to use firm-level data to analyze the dynamics of capital renting. Our primary data source which provides information on rented capital is the company-level panel database of Compustat. The dataset contains financial and economic data on 26,468 publicly listed firms in the United States from 1950 till 2009. However, due to widely documented structural changes in the economy, and in order to achieve a reasonable minimum annual sample size of firms (in this case, around 4,000), we start our investigation only from 1980. This enables us to obtain 261,149 firm-year observations associated with 18,151 firms, of which firms belonging to industry (mainly manufacturing) and to services are about equally present (see Table 3 of the Appendix).

Our rental measure uses rental expenses (variable xrent in Compustat) in the numerator of the following rental share measure, taken from Eisfeldt and Rampini (2009):

$$\text{rental rate}_{it} = \frac{\text{rent}_{it}}{\text{rent}_{it} + (\delta_{it} + i_{it})K_{it}}$$

(2.1)

where the depreciation rate, $\delta_{it}$, is calculated from the reported depreciation, interest rates, $i_{it}$, denote firm-level interest rates on borrowing, and $K_{it}$ is the book value of assets. This measure represents the share of expenditure on capital renting in total capital expenditure, assuming that the book value of capital has the implicit per-period cost of depreciation and interest on borrowing. As such, it provides with a measure of the importance of rented capital in all capital used in production.

In order to mitigate the effect of firm-level noise as well as to appropriately weight observations with capital.
capital expenditures, we use the following aggregate measure in analysis:

\[
\text{rental rate}_t = \frac{\sum_i rent_{it}}{\sum_i (rent_{it} + (\delta_{it} + i_{it})K_{it})}
\]  

(2.2)

This equation (2.2) will be used as an aggregate measure of the rental share in total capital expenditure.\(^6\)

Since our paper is concerned with the cyclicality of renting, it is worth presenting some illustrative time-series evidence on the evolution of renting over time:

Figure 1: Rental shares by firm size (asset quartiles) in US

Figure 1 presents the evolution of the aggregate rental share for four different size categories for the period 1980-2010. The top line refers to firms falling in the smallest size category (defined by the lowest asset quartile, for each year), and the bottom line refers to firms of the largest size category. The gray-shaded areas denote the US recessionary periods as defined by NBER. There are three important messages conveyed by figure 1. Firstly, the share of renting is counter-cyclical, that is, it increases during recessions. Secondly, both the rental share and the volatility of renting decrease in firm size. Thirdly, there is generally an upward trend in the share of renting. The focus of the present paper will be the aggregate study of the counter-cyclicality of renting, and the implications for firm-size and long-run trend will be left for future research.

\(^6\)As an alternative measure, we also use in the denominator total capital expenditures instead of capital services, as in Eisfeldt and Rampini (2009):

\[
\text{rental rate}_t = \frac{\sum_i rent_{it}}{\sum_i (rent_{it} + \text{capital expenditures}_{it})}
\]

The measure called capital expenditures is a direct measure of expanding, upgrading, maintaining the capital stock, instead of inferring them from the reported depreciation and interest payments. This measure yields a very similar pattern to the one defined in 2.2.
To focus more closely on the cyclical behavior of renting, it is worth comparing the cyclical behavior of real GDP and the rental share:

**Figure 2: The cyclical component of rental share and GDP**

![Graph showing the cyclical component of rental share and GDP](image)

Source: Compustat and Penn World Table
Note: The rental share is defined as \( \frac{\text{rent}}{(\text{rent} + (\text{interest rate} + \text{depreciation rate}) \times \text{assets})} \).

In figure 2, the blue columns show the HP-filtered GDP, and black line depicts the HP-filtered overall rental share. The frequency adjustment for annual data was based on Ravn and Uhlig (2002). Figure 2 shows that renting is counter-cyclical, and the degree of counter-cyclicality has become even more pronounced for the second half of the sample period.

The key message of the present paper is that the counter-cyclicality of the rental share naturally arises in a world with financial constraints. The intuition is that in recessions when financial conditions deteriorate, borrowing constraints tighten, and firms have difficulty obtaining credit in order to undertake productive capital investment. In such an environment, the possibility to rent productive capital might become increasingly attractive. This is because, as long as the downpayment for investing in owned capital is higher than paying for the rental service, renting saves cash.

The dual role of capital renting as a production factor as well as a cash-saving device will be a recurring theme of this paper. The idea that capital renting might alleviate liquidity problems is motivated by a strand of corporate finance literature (Opler, Pinkowitz, Stulz, and Williamson, 1999; Holmstrom and Tirole, 1998, 2001) which emphasized the role of liquid financial assets in serving as cushion against potential liquidity and financing problems. We argue that capital renting may also serve as means of increasing liquidity in times of financial distress. To motivate this idea, the following figure provides evidence on the relationship between tightening credit conditions and the increasing share of renting.
Figure 3: The change in rental shares and a measure of credit tightness

Source: Compustat and Senior Loan Officer Survey from the St. Louis FED
Note: The change in rental share is defined as the first difference of \( \frac{\text{rent}}{(\text{rent} + (\text{interest rate} + \text{depreciation rate}) \times \text{assets})} \)

Change in credit tightness is the Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans Large and Medium Firms from the Senior Loan Officer Survey.

Figure 3 shows that there is a close relationship between deteriorating credit conditions (measured by the Senior Loan Officer Survey of the FED), and the increasing share of renting. In an environment of financial distress, the possibility to rent may serve as a means to alleviate borrowing constraints and liquidity problems by saving cash. This is the key intuition that motivates the development of the theoretical model presented in the next section.

3 Model

To conceptionalize the intuitive channel suggested by the empirical evidence, we build the following model. Consider a discrete time, infinite horizon economy, populated by two sectors of risk-averse agents of equal mass: households and entrepreneurs. There are two types of commodities: goods and loanable bonds. Inputs for production consist of capital only whose accumulation takes place in both sectors. There are two important distinctions between the two sectors. Firstly, only the entrepreneur undertakes production, and the household lives off from his financial income. Secondly, only the entrepreneur is subject to credit constraints, whereas the household has unlimited access to credit markets to smooth his consumption. Output is homogeneous, and can be used for either consumption or capital investment by both agents. Both the representative household’s and the entrepreneur’s utility is a function of the consumption good. Furthermore, households are more patient and have a higher subjective discount rate compared to the entrepreneurs.
3.1 Households

The representative household has the following utility maximization problem:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \log C^H_t$$  \hspace{1cm} (3.1)

where $\beta \in (0, 1)$ is the subjective discount factor, and $C^H_t$ denotes the household’s consumption at time $t$. The household is risk-averse, captured in the logarithmic utility function. This will ensure that the household wants to smooth consumption over time, which will be crucial for generating counter-cyclical renting. The flow of funds constraint is given by:

$$C^H_t + \frac{S_t}{R_t} + I^R_t = S_{t-1} + R^R_{t-1}K^R_{t-1}$$  \hspace{1cm} (3.2)

Equation (3.2) shows that the household’s current expenditure is used for consuming, $C_t$, buying a one-period riskless discount bond, $\frac{S_t}{R_t}$, and undertaking capital investment, $I^R$. The expenditure is financed by selling the discount bond purchased in the previous period and by acquiring the previous period’s rental fee, $R^K_{t-1}$, after previous period’s rental service, $K^R_{t-1}$. The accumulation process of the household’s capital stock is defined as follows:

$$K^R_t = (1 - \delta^R)K^R_{t-1} + I^R_t$$  \hspace{1cm} (3.3)

where $\delta^R$ denotes the depreciation rate of the investment good produced by the household.

3.2 Entrepreneurs

The representative entrepreneur has a decreasing-returns-to-scale, production technology that uses capital to produce a homogeneous good $Y_t$ according to:

$$Y_t = Z_t K^\alpha_{t-1}$$  \hspace{1cm} (3.4)

where $Z_t$ denotes the technology shock, $\alpha$ is the scale parameter, and $K_t$ is the composite capital input. Productive capital takes one period before it can be used in production. The technology shock, $Z_t$, follows a first-order autoregressive process:

$$\ln Z_t = \rho_z \ln Z_{t-1} + \xi^z_t$$  \hspace{1cm} (3.5)

where $\rho_z$ measures the degree of persistence of the technology shock, and $\xi^z_t$ is a white noise term with a standard deviation of $\sigma^2_z$.

Composite capital consists of both owned capital, $K^O$, and the capital the entrepreneur rents from the household, $K^R$. Regarding the composition of productive capital, the following assumption is made:
**Assumption 1** There is an imperfect degree of substitutability in production between rented and owned capital.

\[ K_t = \left[ \omega \left( K_t^O \right)^\frac{\varepsilon - 1}{\varepsilon} + (1 - \omega) \left( K_t^R \right)^\frac{\varepsilon - 1}{\varepsilon} \right]^\frac{1}{\varepsilon - 1} \]  

(3.6)

where \( K_t^O \) is capital owned by the entrepreneur, and \( K_t^R \) denotes capital rented from the household. \( \omega \) captures the share of the two types of capital in the composite capital, and \( \varepsilon \) denotes the degree of substitution between owned and rented capital. The motivation behind assumption 1 is both theoretical and empirical. As shown later explicitly, perfect degree of substitutability would lead to a corner solution. That is, depending on the relative cost of the two types of capital, all capital would be either rented or owned. Introducing some degree of complementarity helps eliminate corner solutions, leads to a unique determination of the share of renting in production, and is also empirically justified. \(^7\)

To introduce financial frictions into our model, we follow Kiyotaki and Moore (1997). We assume that there is a limited contract enforcement problem giving rise to borrowing constraints faced by the entrepreneur:

**Assumption 2** There is a limit on the amount of collateralised borrowing the entrepreneur can obtain, and only owned capital can be used as collateral.

\[ B_t \leq \theta K_t^O \]  

(3.7)

The intuition behind assumption 2 is based on the idea of the “inalienability” of human capital, as in Hart and Moore (1994). The production technology is specific to each entrepreneur, which increases his bargaining power against lenders. Once an entrepreneur invested at date \( t \), only he has the skill to obtain the return on his investment at time \( t + 1 \), whereas others can obtain only \( \theta \) fraction of the return, which can be thought of as the collateral. More specifically, if the entrepreneur fails to fulfill his debt obligation, \( B_t \), then the lender (the household) can repossess only the collateral, which is \( \theta \) fraction of the entrepreneur’s owned capital stock.

It is important to note that only the entrepreneurial sector where the accumulation of owned capital takes place is subject to borrowing constraints, and the household sector where the rented capital is accumulated is not. \(^8\)

The maximization problem of the entrepreneur can be written as:

\[ \max E_t \sum_{t=0}^{\infty} \gamma^t \log C_t^E \]  

(3.8)

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\(^7\)One may argue that certain capital goods needed in the production process are more easily rented than owned. For instance, trucks used for transporting goods are easily leasable, whereas the production equipment of a factory where the goods are produced can be firm- or plant specific, therefore may be hard to rent. Based on our dataset, we also find empirical evidence at both firm- and industry-level for a considerable degree of complementarity between rented and owned capital, with the estimated values for \( \varepsilon \) being approximately \( 1 - 1.5 \). See section 7.5.

\(^8\)This aspect of our model is similar to the set-up of Kiyotaki and Moore (1997). However, production in our model, unlike in Kiyotaki and Moore (1997), is undertaken only by the entrepreneur, which makes the analysis of changes in the composition of rented and owned capital in production more tractable. Our assumption that the rental sector (the household) is not subject to borrowing constraints is the limiting case of a more general set-up whereby capital accumulation in all sectors of the economy is subject to different degrees of constrained borrowing, as in Matsuyama, 2007, and these constraints are assumed to be less severe in the rental capital sector. Such generalization would not change our theoretical results.
where $C^E_t$ denotes entrepreneurial consumption, and $\gamma$ is the subjective discount factor of the entrepreneur. Note that this formulation can also be interpreted as maximizing the dividend stream, in a way that those payments are to be smoothed over time.

**Assumption 3 $\gamma < \beta$**

Assumption 3 follows Kiyotaki and Moore (1997) and Iacoviello (2005), and it ensures that the borrowing constraint (3.7) is binding, and entrepreneurs do not postpone consumption in order to accumulate capital and self-finance production, but they always borrow up to the limit. The maximization problem (3.8) is subject to the technology constraint (3.4), the borrowing constraint (3.7), and the following flow of funds and investment constraints:

$$C^E_t + I^O_t + R^R_{t-1}K^R_{t-1} + B_{t-1} = Y_t + \frac{B_t}{R_t}$$  \hspace{1cm} (3.9)

$$K^O_t = (1 - \delta^O) K^O_{t-1} + I^O_t$$ \hspace{1cm} (3.10)

where equation (3.9) shows that the entrepreneur uses his income from production, $Y_t$, and borrowing from the issuance of a one-period discount bond, $\frac{B_t}{R_t}$, to finance consumption, $C^E_t$, investment into owned capital, $I^O_t$, capital renting expenditure, $R^R_{t-1}K^R_{t-1}$, and previous debt obligation, $B_{t-1}$. Equation (3.10) defines the accumulation of the entrepreneurial capital stock, that depreciates at rate $\delta^O$.

**Assumption 4 $\delta^O < \delta^R$**

Assumption 4 embodies the problem of separation of ownership and control associated with rented capital. The difference between $\delta^O$ and $\delta^R$ can be thought of as a premium associated with the higher monitoring cost of rented capital. Assumption 4 captures the disadvantages of rented capital related to a faster depreciation rate in production and its more costly maintenance. This approach follows the finance literature on leasing, as in Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2010a). It is important to note that in our model, where rented and owned capital are assumed to be imperfect substitutes, assumption 4 is not essential for our main theoretical results, regarding the counter-cyclically of rental share, to hold. The adoption of assumption 4 is mainly motivated by our empirical evidence on the higher depreciation rate of rented capital, and it also serves the purpose of matching the empirical moments in the data better.

To sum up, assumption 1 is needed to avoid corner solutions, and is also supported by empirical evidence. Assumption 2 introduces borrowing constraints which gives rise to the main advantage of renting over owning, namely that rental capital accumulation is not subject to credit frictions and related liquidity problems, unlike the accumulation of owned capital. Assumption 3 ensures that the borrowing constraint always binds. Assumption 4 captures the major disadvantage of renting, namely that rented capital is a less efficient production factor than owned capital.
3.3 Equilibrium

In the absence of shocks, the model has a unique stationary equilibrium in which entrepreneurs borrow up to the limit, and all markets clear. The goods market clear condition implies that:

\[ C_t^H + C_t^E + I_t^R + I_t^O = Y_t \]  \hspace{1cm} (3.11)

The bond market clearing condition implies that:

\[ S_t = B_t \]  \hspace{1cm} (3.12)

The competitive equilibrium is an allocation \{Y_t, C_t^H, C_t^E, K_t^R, K_t^O, I_t^R, I_t^O, S_t, B_t\}_{t=0}^{\infty} together with the sequence of prices \{R_t, R_t^R, \psi_t\} such that the allocation solves the optimization problems for the household and the entrepreneur, and all markets clear. The steady-state of the model is derived in appendix 7.2.1, and the non-linear equilibrium conditions are found in appendix 7.2.2.

3.4 The rental share in steady-state

The optimality conditions imply that the steady-state ratio of rented and owned capital is uniquely determined by the preference parameters (\(\beta\) and \(\gamma\)), the capital parameters (\(\delta^R, \delta^O, \varepsilon\) and \(\omega\)) and the tightness of the borrowing constraint (\(\theta\)). Given realistic parameter values, the renting-owning ratio in steady-state is decreasing in (i) \(\theta\), (ii) \(\gamma\), (iii) \(\delta^R\), and (iv) \(\varepsilon\), as shown in:

\[ \frac{K_t^R}{K_t^O} = \left( \frac{\frac{1}{\gamma} - (1 - \delta^O) - \theta(\beta - \gamma)}{\frac{1}{\beta} - (1 - \delta^R)} \right) \varepsilon \left( \frac{1 - \omega}{\omega} \right) \]  \hspace{1cm} (3.13)

Equation (3.13) is derived from the steady-state solution of the model presented in the appendix. Intuitively, (3.13) suggests that the tighter the borrowing constraint the entrepreneur faces, represented by \(\theta\), the less capital will be owned, and the more capital will be rented. In addition, the lower the entrepreneur’s subjective discount rate (\(\gamma\)), the more impatient he becomes and the more capital he wants to rent. Also, the higher the depreciation rate of rented capital (\(\delta^R\)), the higher its user cost. As a result of the increasing costliness of renting, its share compared to owned capital falls. Finally, the higher the complementarity between owned and rented capital, the lower the share of renting\(^9\).

In case of perfect substitutability (\(\varepsilon = \infty\)), the share of renting becomes indeterminate, and equation (3.13) collapses into the following corner solution:

\[ \frac{1}{\beta} + \delta^R \geq \frac{1}{\gamma} + \delta^O - \theta(\beta - \gamma) \]  \hspace{1cm} (3.14)

where the left-hand-side of equation (3.14) denotes the return on renting, whereas the right-hand-side rep-

\(^9\)Note that our empirical measure (2.2) is a strictly monotonous function of the expression (3.13).
represents the return on owning. Depending on the parameter values, all capital in the economy will be either rented or owned.

### 3.5 The rental share over the business cycle

This section shows that the presence of borrowing constraints causes the share of capital renting to be countercyclical. To prove this claim, let us first analyze our model without borrowing constraints and without the possibility to rent.\(^\text{10}\) In this case, our model collapses to the standard neoclassical model of investment:

\[
1 + MPK^O_{t+1} - \delta^O = R_t
\]  
\text{(3.15)}

Equation (3.15) is the standard arbitrage condition stating that the return on physical capital investment and the return on financial investment must always equal. Introducing borrowing constraints into the standard neoclassical model yields the following modified condition:

\[
1 + MPK^O_{t+1} - \delta^O = R_t \left( \frac{1 - \theta \psi_t}{1 - R_t \psi_t} \right)
\]  
\text{(3.16)}

Equation (3.16) shows that the presence of borrowing constraints introduces a wedge between the return on financial investment and the return on physical investment. This wedge, \(\left( \frac{1 - \theta \psi_t}{1 - R_t \psi_t} \right)\), can also be interpreted as a premium on the cost of holding one unit of owned capital compared to holding a financial asset. The behavior of the premium will be explained in more detail below.

Allowing for the possibility to rent productive capital offers the households the option to invest in renting services as opposed to investing in financial assets only. In the absence of borrowing constraints, the equilibrium condition for the returns on owned and rented capital is written as follows:

\[
1 + MPK^O_{t+1} - \delta^O = 1 + MPK^R_{t+1} - \delta^R
\]  
\text{(3.17)}

Equation (3.17) shows that in equilibrium, the returns on both types of capital always equal. It is important to note that the two marginal products of capital\(^\text{11}\) differ only in the degree of elasticity of substitution, \(\varepsilon\). In the case of perfect substitutability, \(\varepsilon = \infty\), the two marginal products of capital would be equal, \(MPK^O_{t+1} = MPK^R_{t+1}\). This would make the return on owned capital be always higher than the return on rented capital because of the higher depreciation rate of rented capital. This would give rise to a corner solution leading to an equilibrium, where all productive capital would be owned by the entrepreneur, and the household would invest all his savings in financial assets.

The presence of borrowing constraints introduces the premium as a wedge between the return on owned and rented capital:

\(^{10}\)All the different scenarios will be analyzed under the assumption of perfect foresight.

\(^{11}\)The definition of the marginal product of owned capital is: \(MPK^O_t = \alpha Z_t K_{t-1}^{\alpha - (\varepsilon - 1)/\varepsilon} \omega^{\varepsilon/\varepsilon - 1} \left( K^O_{t-1} \right)^{-\frac{1}{\varepsilon}}\); whereas the definition of the marginal product of rented capital is \(MPK^R_t = \alpha Z_t K_{t-1}^{\alpha - (\varepsilon - 1)/\varepsilon} (1 - \omega)^{\frac{1}{\varepsilon}} \left( K^R_{t-1} \right)^{-\frac{1}{\varepsilon}}\).
\[(1 + MPK_{t+1}^O - \delta^O) = (1 + MPK_{t+1}^R - \delta^R) \left( \frac{1 - \theta \psi_t}{1 - R_t \psi_t} \right) \] (3.18)

Equation (3.18) is the heart of our model, which is crucial to understand the counter-cyclical behavior of the rental share. As in equation (3.16), the presence of borrowing constraints gives rise to a wedge between the return on owned and rented capital. The behavior of this wedge, which we call premium, is largely dependent on the shadow price of the borrowing constraint, \( \psi_t \). Understanding the behavior of \( \psi_t \) in response to shocks, is crucial for understanding the reaction of the premium. For example, assume the economy is hit by a negative technology shock, after which the entrepreneur’s production and wealth are directly reduced. Because of the logarithmic specification of his utility function, the entrepreneur is always interested in smoothing his consumption by accessing the credit markets. However, his ability to access the credit markets is limited by the presence of borrowing constraints. The larger the negative technology shock, the higher his demand for credit to smooth consumption, and the more severely he is hit by the borrowing constraint which is associated with a higher shadow price, \( \psi_t \) leading to a rise in the premium.\(^{12}\) As a result, the return on and the cost of owned capital increases relative to the return on and the cost of rented capital. Because of standard general equilibrium effects, this requires an adjustment of the relative quantities such that the quantity of rented capital must increase relative to the quantity of owned capital.

In order to understand the intuition behind the counter-cyclical behavior of the rental share, one has to understand the trade-off between renting and owning. On the one hand, there is a collateralisability-effect, that is, an extra unit of owned capital increases the entrepreneur’s collateral basis allowing for more debt to be taken out. On the other hand, there is a liquidity-effect, that is, owning an extra unit of capital requires a higher downpayment, whereas renting saves cash. This trade-off is inherent in the premium:

\[
\text{premium}_t = \frac{1 - \theta \psi_t}{1 - R_t \psi_t} \] (3.19)

The numerator of the premium, \( 1 - \theta \psi_t \), captures the implicit benefit of owning relative to that of renting because of the positive collateralisability-effect. Each unit of owned capital allows for a \( \theta \) fraction of extra debt to be taken out. In crises, when the shadow price of the borrowing constraint, \( \psi_t \), increases, the implicit benefit of owning increases. The denominator of the premium, \( 1 - R_t \psi_t \), captures the negative liquidity-effect of owning, which is associated with the difference between the stochastic discount factor of the household, \( D_{t,t+1}^H \), and that of the entrepreneur, \( D_{t,t+1}^E \).\(^{13}\) The intuition is that in the presence of tightening borrowing constraints, the entrepreneur’s marginal utility of current consumption exceeds his marginal utility of tomorrow’s consumption, whereas the household’s marginal utility of current’s consumption continues to be equal to the marginal gain of shifting his consumption into the future. This increases the entrepreneur’s demand for

\(^{12}\)To show that the premium increases in \( \psi_t \), take the following partial derivative: \( \frac{\partial}{\partial \psi_t} \left( 1 - \frac{\theta \psi_t}{1 - R_t \psi_t} \right) = R_t - \theta > 0 \), as \( R_t \), the gross interest rate is always assumed to bigger than \( \theta \).

\(^{13}\)Note, that the liquidity-effect term can be written as \( 1 - R_t \psi_t = \frac{D_{t,t+1}^E}{D_{t,t+1}^H} \), whereas the difference between the two agents’ stochastic discount factors is precisely the shadow price of the borrowing constraint: \( \psi_t = D_{t,t+1}^H - D_{t,t+1}^E \).
current consumption relative to the household, hence increasing the entrepreneur’s relative demand for cash. The difference in how the two agents’ intertemporal consumption profiles change in response to a negative shock in the presence of borrowing constraints is crucial to understand why the entrepreneur wants more purchasing power at time $t$ relative to the household.\footnote{One may think of the two agents’ changing relative cash-needs as a consequence of a shift in the two agents’ time-varying relative impatience. This shift is caused by the entrepreneur’s inability to avoid an immediate drop in his consumption following a negative shock at time $t$ because of the borrowing constraints he faces. (In contrast, the household has the ability to spread this initial consumption loss for the subsequent periods because of his access to credit markets.) As a result, following the initial realization of welfare loss from period $t+1$ onwards, the entrepreneur perceives the future in a relatively “brighter” and more “optimistic” way leading to a higher level of impatience relative to the household.} The entrepreneur, upon deciding on how to spend his present purchasing power on getting hold productive capital, can choose between paying a fee, $R^H_t$, to rent one unit of capital, and offering a sum, $1 - \frac{\theta}{R_t}$, as downpayment to borrow an amount, $\frac{\theta}{R_t}$, to buy one unit of capital. It can be shown that the premium is directly related to the value of the total cash held by both the household and the entrepreneur. To illustrate this, equation (3.19) is rewritten to obtain the following decomposition of the premium:

$$\text{premium}_t = \frac{D^H_{t+1}}{D^E_{t+1}} \left( 1 - \frac{\theta}{R_t} \right) + \frac{\theta}{R_t}$$ \hspace{1cm} (3.20)

Equation (3.20) shows that the premium can be decomposed into two parts. On the one hand, the premium is related to the changing discounted value of the cash-amount, $1 - \frac{\theta}{R_t}$, that the entrepreneur spends on the downpayment. On the other hand, the premium is related to the cash-amount the household decides to lend to the entrepreneur, $\frac{\theta}{R_t}$. The increasing premium and the increasing liquidity-need of the economy following a negative shock must be associated with an increase in the valuation of the cash-amount spent by the entrepreneur on the downpayment.

At the margin, the entrepreneur allocates his purchasing power on paying rental fees and spending on downpayments such that the returns on the two types of cash-investments should be the same. In times of increasing liquidity-needs, there is an increase in the net market-value of the cash-amount spent on the downpayment, which must be associated with an increase in the return on buying an extra unit of capital relative to renting it. To put it simply, the entrepreneur wants to choose the capital technology that is less cash-intensive, hence he substitutes rented capital for owned capital.\footnote{Note, that the consumption-smoothing motive behind the liquidity effect is a well-known phenomenon in the finance literature. See for example Holmstrom and Tirole (1998, 2001); Eisdorf (2007).}

Note that the leverage ratio, $\theta$ plays an important role in the mechanism described above. Ceteris paribus, the higher the $\theta$, the higher collateralisability-effect relative to the liquidity-effect. This lowers the response of the premium to a rise in the shadow price of the borrowing constraint, $\psi_t$, hence lowers the wedge between the returns on owned and rented capital. Therefore, for a given $\psi_t$, the counter-cyclicality of the rental share should decrease in $\theta$. However, it would be wrong to conclude that the counter-cyclicality of the rental share in general decreases in $\theta$, since increasing $\theta$ fundamentally changes the leverage structure of the economy, which then substantially changes the response of $\psi_t$ to shocks. In fact, our simulation results will show that
the counter-cyclicality of the rental share increases in $\theta$.

### 3.6 Model Dynamics

The calibrated, quarterly parameter values are described in appendix 7.4. Following Krusell, Smith, and Jr. (1998), Iacoviello (2005) and Monacelli (2009), the subjective discount factors of the household and the entrepreneur are $\beta = 0.99$ and $\gamma = 0.98$ respectively. Similar to Caggese (2007), the returns-to-scale parameter is set higher, $\alpha = 0.6$, than in models with labour markets. The share of renting in composite capital is set $\omega = 0.8$, which is calibrated to match the steady-state value of rental share suggested by the model with the value (around 25%) observed in the data. The degree of elasticity between rented and owned capital is set higher, $\varepsilon = 1.5$. The depreciation rate of owned capital is set higher, $\delta^O = 0.02$, than the depreciation rate of rented capital, $\delta^R = 0.03$. These values are based on our firm-level dataset. The tightness of the borrowing constraint is $\theta = 0.6$, similar to Caggese (2007) and Monacelli (2009).

This section shows impulses responses of a set of key variables following a negative, one standard deviation shock to technology, $Z_t$. The persistence of the technology shock is assumed to be relatively high: $\rho_z = 0.91$, as standard in the literature. Following Smets and Wouters (2007), the standard deviation of the technology shock is set 0.45.

The impulse responses are obtained after taking first-order linear approximation of the model described above. The full description of the log-linearized model can be found in appendix 7.2.3.
Figure 4: The effects of a negative technology shock in the benchmark model

Note: The vertical axis shows percentage deviations from the steady state values in the benchmark model. The horizontal axis is in quarters.

Figure 4 shows that following a persistent technology shock, output drops immediately and remains below its steady-state level persistently. Productive composite capital and owned capital fall after the shock, and they peak at a negative 0.5% below their respective steady-state levels. In contrast, rented capital increases and stays above its steady-state level, whereas the share of renting exhibits a similar but more persistent behavior.\textsuperscript{16} As discussed above, the counter-cyclicality of rental share is predicted by equation (3.18), because the tightening of the borrowing constraint associated with an increase in its shadow price, \( \psi \), leads to a rising premium which in turn increases the relative cost of owned capital compared to the cost of rented capital. The presence of the borrowing constraints is demonstrated by the consumption responses: while the households' consumption shows a gradual decline, the entrepreneur’s consumption exhibits an immediate drop because of limited access to credit markets, hence limited consumption smoothing.

3.7 Business cycle statistics

The model with aggregate productivity shocks can explain a number of business cycle statistics observed in historical US data. Table 1 presents the simulation results under the benchmark parametrization. The top panel of the table reports on the key steady-state ratios observed in the data and implied by the model. The

\textsuperscript{16}The definition of the rental share is the same as the one used during the empirical analysis explained by equation (2.2).
steady-state values of all the big ratios such as the capital stock, consumption and investment as a share of GDP are well approximated by the benchmark model.

The panel in the middle of table 1 reports on the annual standard deviations and the correlations with output of four main variables. The somewhat unusual exercise of reporting business cycle statistics at annual frequency is motivated by the fact that the cyclical behavior of the rental share can not be observed at higher frequency. Both models perform well in matching the standard deviation of aggregate output and consumption; however they do not generate sufficient degree of investment volatility as observed in the data. The key variable in this paper is the rental share that is observed to have a largely negative correlation with output (-0.76) reflecting on the strong counter-cyclicality of the rental share. The simulated benchmark model yields a value (-0.82) close to that observed in the data.

The panel at the bottom reports on the results for quarterly frequency. Again, the benchmark model performs well in matching the main business cycle moments except the standard deviation of aggregate investment.

Table 1: Business Cycle Statistics

<table>
<thead>
<tr>
<th>Variables</th>
<th>Average over 1980-2009</th>
<th>Model SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>K/Y *</td>
<td>3.367</td>
<td>3.463</td>
</tr>
<tr>
<td>C/Y</td>
<td>0.674</td>
<td>0.699</td>
</tr>
<tr>
<td>I/Y</td>
<td>0.211</td>
<td>0.301</td>
</tr>
</tbody>
</table>

Annual

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data (1980-2009)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>St.dev.</td>
<td>Correlation with output</td>
</tr>
<tr>
<td>Output</td>
<td>1.272</td>
<td>1.000</td>
</tr>
<tr>
<td>Rental share **</td>
<td>0.845</td>
<td>-0.760</td>
</tr>
<tr>
<td>Investment</td>
<td>4.323</td>
<td>0.949</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.991</td>
<td>0.903</td>
</tr>
</tbody>
</table>

Quarterly

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data (1980-2009)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>St.dev.</td>
<td>Correlation with output</td>
</tr>
<tr>
<td>Output</td>
<td>1.415</td>
<td>1.000</td>
</tr>
<tr>
<td>Rental share</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Investment</td>
<td>4.797</td>
<td>0.934</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.225</td>
<td>0.858</td>
</tr>
</tbody>
</table>

Notes:
* K/Y is taken over the average 2003-2009.
** Rental share is taken from Compustat annual data, and is not available at quarterly frequency.
The rental share is calculated as follows:
rental share = rental expenses / (rental expenses + depreciation + interest rate * fixed tangible assets).
The denominator captures total capital expenditures, imputed by using book value depreciation rates and interest rates. Interest rates are short term borrowing rates as reported by the company. The formula closely follows Eisfeldt and Ramnis (2009).
4 The role of asset prices

In the benchmark model, only one homogeneous good was considered, and assets with fixed supply have been excluded from the analysis. As a result, the borrowing constraint lead to a pure quantity effect, and the traditional asset-price channels have been ignored. Hence the benchmark model will now be extended by introducing an asset with fixed supply: land. This enables us to study the interaction between the value of the fixed asset and the cyclicality of the rental share. A fraction of the fixed land supply is owned by the households, and the remaining fraction is owned by the entrepreneur.

4.1 Households

The representative household has the same utility function as in (3.1). Compared to the benchmark model, the household’s budget constraint is modified by introducing investment expenditures on land, and also by differentiating between the capital he accumulates and the capital he rents out:

\[ C_t + \frac{S_t}{R_t} + I_t^H + q_t \left( L_t^H - L_{t-1}^H \right) = S_{t-1} + R_t^R K_{t-1}^R \]  

(4.1)

where \( L_t^H \) denotes household land with price \( q_t \), and \( I_t^H \) is the household’s capital investment, which follows a similar accumulation process as described in equation (3.3), and can be written as:

\[ K_t^H = (1 - \delta^R) K_{t-1}^H + I_t^H \]  

(4.2)

where \( K_t^H \) denotes the capital stock of the household. A major difference between the modified and the benchmark model is that the capital the household rents out is now a composite asset consisting of both household land, \( L_t^H \), and household capital \( K_t^H \). Following Kiyotaki, Michaelides, and Nikolov (2011), the composite rented asset is produced using a constant returns to scale technology:

\[ K_t^R = \left( L_t^H \right)^{1-\phi} \left( K_t^H \right)^{\phi} \]  

(4.3)

where \((1 - \phi)\) measures the importance of household land in the composition of rented capital.

4.2 Entrepreneurs

The representative entrepreneur has the same utility as in (3.8), but his budget constraint is modified as follows:

\[ C_t^e + I_t^E + q_t \left( L_t^E - L_{t-1}^E \right) + R_t^R K_{t-1}^R + B_{t-1} = Y_t + \frac{B_t}{R_t} \]  

(4.4)

where \( L_t^E \) denotes household land with price \( q_t \), whereas \( I_t^E \) is entrepreneurial capital investment which follows a similar accumulation process as described in equation (3.10), and written as:
\[ K_t^E = (1 - \delta^O) K_{t-1}^E + I_t^E \]  

where \( K_t^E \) denotes the capital stock of the entrepreneur. A major difference between the modified and the benchmark model is that the capital the entrepreneur owns is now a composite asset consisting of both entrepreneurial land, \( L_t^E \), and entrepreneurial capital \( K_t^E \). The composite owned asset is written as follows:

\[ K_t^O = (L_t^E)^{1-\phi} (K_t^E)^{\phi} \]  

The modified definitions of rented and owned capital and fluctuations in the price of the fixed assets ensure that capital goods are no longer identical to consumption goods, hence their price is no longer 1. In fact, the price of fixed asset is directly related to the price of the two types of composite capital, denoted by \( q_t^\star \).

\[ q_t^\star = q_t^{1-\phi} \left( \frac{1}{1-\phi} \right) \left( \frac{1-\phi}{\phi} \right)^\phi \]  

Equation (4.7) is the key step to connect the prices of the fixed asset with the prices of the owned and rented capital, hence introducing the asset-price channel into the benchmark model. Movements in the prices of the fixed asset not only change the value of owned capital, but they also change the value of the collateral that the entrepreneur can offer in order to borrow. The borrowing constraint defined by equation (3.7) is now extended with the an asset-price term:

\[ B_t \leq \theta_t q_t^\star K_t^O \]  

Note, that equation (4.8) also differs from equation (3.7) in the definition of the collateral. In the modified model, the entrepreneur can collateralize his own land, \( L_t^E \), in addition to collateralizing the productive capital he accumulates, \( K_t^E \), and the collateral base is determined by the Cobb-Douglas function (4.6).

4.3 The key properties of the extended model

The presence of asset prices following the introduction of land into the benchmark model gives rise to two different types of premia that are related to owning as opposed to renting entrepreneurial capital and land. A key difference between the premium of the benchmark model and the premia in the extended model is that the presence of asset prices changes the collateralisability-effect while leaving the liquidity-effect unchanged.

This can be seen by deriving the equilibrium condition, similar to equation (3.18) in the benchmark model, which connects the return on entrepreneurial and household capital in the presence of asset prices:

\[ (1 + MP K_{t+1}^H - \delta^H) = (1 + MP K_{t+1}^E - \delta^E) \left( \frac{1 - \theta_t^\psi_t q_t^{MP K_{t+1}^E}}{1 - R_t^\psi_t} \right) \]  

\[ (4.9) \]
where $MPK_{t+1}^H$ denotes the next period marginal product of household capital, $K_t^H$, that is part of the rented capital package, $K_t^R$; and $MPK_{t+1}^E$ is the next period marginal product of entrepreneurial capital, $K_t^E$, that is part of the owned capital package, $K_t^O$. Equation (4.9) shows that the premium on owned entrepreneurial capital is now modified by movements in $q_t^*$, that is, the price of the owned capital package, $K_t^O$. Following the discussion related to equation (3.19), the implicit cost of owning as opposed to renting one unit of capital is equal to 1 in the absence of borrowing constraints. However, in the presence of borrowing constraints this implicit cost, as described by the the term $1 - \theta \psi_t q_t^* \frac{MPK_{t+1}^E}{MPK_{t+1}^O}$, is no longer 1, but less by an amount associated with the benefit obtained by using $\theta$ fraction of the one unit of owned capital as collateral for further borrowing. A key insight is that collateralisability of one unit of newly obtained entrepreneurial capital, $K_t^E$, now also depends on movements in the price, $q_t^*$, of the owned capital package, $K_t^O$. When asset prices fall, the collateralisability-effect associated with owning is reduced. However, it is important to note that the impact of a fall in $q_t^*$ on the decreasing collateral value is adjusted by the term, $\frac{MPK_{t+1}^E}{MPK_{t+1}^O}$, denoting the contribution of one unit of entrepreneurial capital to the whole capital package in production use.

Similarly, the equilibrium conditions of the land market can be used to express the collateralisability-effect and the liquidity-effect corresponding to owning as opposed to renting one unit of land at price $q_t$:

$$\left(MPL_{t+1}^H + q_{t+1}\right) = \left(MPL_{t+1}^E + q_{t+1}\right) \left(1 - \theta \psi_t q_t^* \frac{MPL_{t+1}^E}{q_t \frac{MPK_{t+1}^E}{MPK_{t+1}^O}} \frac{1 - R_t \psi_t}{1 - R_t \psi_t}\right)$$

(4.10)

where $MPL_{t+1}^H$ denotes the next period marginal product of household land, $L_t^H$, that is part of the rented capital package, $K_t^R$; and $MPL_{t+1}^E$ is the next period marginal product of entrepreneurial land, $L_t^E$, that is part of the owned capital package, $K_t^O$. Equation (4.10) connects the return on renting one unit of household land, $L_t^H$, to the return on owning one unit of entrepreneurial land, $L_t^E$. For both types of land, the implied return on one unit of land is associated with the marginal product, $MPL$, and the selling of the land next period at price $q_{t+1}$. In addition, the premium captures the liquidity-effect and the collateral effect related to owning the land as opposed to renting it. The dynamics of the premium is now less obvious since it depends on how the price and the next period marginal product of the land moves in relation to the price and the next period marginal product of the owned capital package. To understand the dynamics of the model, simulation techniques are applied.
Figure 5: The effects of a negative technology shock in the model with asset prices

Figure 5 shows the impulse responses of the model economy with asset prices following a $-0.45$ standard deviation shock to total factor productivity. Similar to figure 4, the negative technology shock causes an immediate drop in output and the total as well as the owned capital stock inducing a reduction in the collateral base. The reduction of the owned capital is associated with a fall in the entrepreneurial land and the land price. Falling land prices in turn imply that the value of the owned capital package also falls leading to a further shrinkage of the collateral base. As a result, the shadow price of the borrowing constraints, $\psi_t$, increases more in response than in the benchmark model without asset prices. Since the positive collateralisability-effect associated with owning is reduced in the presence of asset prices, the premium increases by more making capital rental share more counter-cyclical than in the model without asset prices.$^{17}$

5 The role of renting in crises

5.1 Model Implications

To understand the importance of renting in the presence of borrowing constraints, we show how the propagation of shocks changes by the presence of the possibility to rent. To illustrate this, we first analyse the impact of a negative technology shock in three different model specifications. The first model is the benchmark model

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$^{17}$These results are confirmed by the analysis of business cycles statistics, whereby the correlation value of renting with output is more negative than in the model without asset prices. These results are available upon request.
explained in section 3 without the possibility to rent\textsuperscript{18}; the second is the benchmark model with renting, and the third model is the one extended with asset prices as explained in 4.

Figure 6: The effects of a negative technology shock in three different model specifications

Figure 6 shows accumulated impulse responses of three key variables in the three different model specifications following a $-0.45$ standard deviation shock to total factor productivity. The effects on output are almost identical suggesting that the presence of renting does not change the propagation of technology shocks. Similarly, the impulses are not substantially different for total capital and the shadow price of the borrowing constraints, $\psi_t$.

Up to now, the main source of uncertainty has been exogenous shocks to the production process and all other shocks have been ignored. However, the recent financial crisis has generated substantial interest in studying shocks generated in the financial system. We therefore extend our analysis by assessing whether the presence of the renting sector substantially changes the response of the model economy to an exogenous change in the leverage ratio, $\theta$, for which we define the following AR(1) process:

$$\ln \theta_t = (1 - \rho_f) \ln \theta + \rho_f \ln \theta_{t-1} + \xi_f^t$$ (5.1)

where $\rho_f$ measures the degree of persistence of the collateral shock, and $\xi_f^t$ is a white noise term. Intuitively, a negative shock to $\theta_t$ can be thought of as an exogenous loss in the degree of trust in the economy leading to deteriorating financial market conditions, reduced lending activities and falling output as presented powerfully in Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). We argue that the presence of a renting sector that is not subject to borrowing constraints can substantially mitigate the adverse effects of a negative collateral shock. To illustrate this, we analyze the impact of a negative collateral shock in the three different model specifications.

\textsuperscript{18}Shutting down the renting sector is done by disallowing the household to accumulate capital. As a result, his only source of income becomes the interest payments received from the entrepreneur.
Figure 7: The effects of a negative collateral shock in three different model specifications

![Graph showing the effects of a negative collateral shock](image)

Notes: The figures represent accumulated impulse responses. The vertical axis shows in percentage deviations from steady-state, and the horizontal axis is in quarters.

Figure 7 shows accumulated impulse responses of four key variables in the three different model specifications following a $-0.45$ standard deviation shock to $\theta_t$. The key message of this figure is that the absence of the possibility to rent substantially amplifies the adverse impact of a negative collateral shock on output. More specifically, in a world without renting the accumulated output loss after 2 years following a negative collateral shock amount to more than 1%, whereas in the benchmark model this is about 0.3%, and in the model with asset prices, the output loss is only about 0.18%. The results are similar for the total capital stock whose accumulated loss 2 years after the realization of the shock amounts to about 2.5% compared to a loss of 0.8% in the benchmark model and about 0.5% in the model with asset prices. The intuition behind the different performance of the models is illustrated by the right panel of figure 7 which shows the response of the shadow price of the borrowing constraints in the three different model economies. The results suggest that the presence of the possibility to rent substantially mitigates problems associated with tightening borrowing constraints.

5.2 Empirical Evidence

In order to test for the prediction that a financial shock will have a smaller impact when rental possibilities are present, we collected cross-country longitudinal data with the size of the rental sector as a proxy for the importance of renting in the economy.\textsuperscript{19}

The questions we address is the following: does the presence of a larger rental sector lead to a smaller reaction of aggregate GDP in response to a financial shock. "Financial shock" is a difficult concept to measure, and as a first approximation, we use the change in private domestic credit to GDP\textsuperscript{20}, as it captures a large fraction of the variation in credit conditions (i.e. a change in the collateral multiplier, represented by $\theta$ in our model). Even though it is normalized by overall GDP, it may be still affected by changes in credit demand.

\textsuperscript{19}The size of the rental sector is measured as the value added of industry 71 in NACE rev1.1 (Renting of machinery and equipment without operator and of personal and household goods) over total economy value added or total economy investment (gross fixed capital formation). The source of the data is the OECD’s STructural ANalysis Database (STAN) which contains annual data on nominal 2-digit value added for 28 countries and from 1970 to 2009.

\textsuperscript{20}source: World Development Indicators (WDI), WorldBank
which can have a simultaneous effect on GDP. In order to control for such endogeneity, we instrument the financial shock by its lagged value. The equation we are estimating is the following:

\[
d\log(Y_{ct}) = \eta_c + \beta_0 + \beta_1 d\log(Credit_{ct}/Y_{ct}) + \beta_2 RentalSector_{t-1,t-5} \\
\beta_3 d\log(Credit_{ct}/Y_{ct}) RentalSector_{t-1,t-5} + \beta_4 \log(GDP/capita)_{t-1,t-5} + \varepsilon_{ct}
\] (5.2)

where \(\eta_c\) is the country fixed-effect and \(\varepsilon_{ct}\) is white noise. The coefficient \(\beta_3\) is of central importance. It is the interaction term of the change in credit and the size of the rental sector which tells us by how much the economy is affected by a financial shock if it has a larger rental sector.

Table 2: The effect of a financial shock is mitigated by having a larger renting sector

<table>
<thead>
<tr>
<th>OLS on cross-country panel</th>
<th>Coef.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>dLog(GDP)</td>
<td>0.025</td>
<td>0.000</td>
</tr>
<tr>
<td>dLog(credit/GDP)</td>
<td>0.010</td>
<td>0.980</td>
</tr>
<tr>
<td>rental_supply * dLog(credit/GDP)</td>
<td>-5.295</td>
<td>0.025</td>
</tr>
<tr>
<td>log(GDP/capita)</td>
<td>-0.017</td>
<td>0.000</td>
</tr>
<tr>
<td>constant</td>
<td>0.199</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Total number of observations: 462
Number of countries: 28
R-squared: 0.161

Note: Rental share is measured as five-year average in t-1 to t-5. log(GDP/capita) is measured as five-year average in t-1 to t-5, using GDP at PPP.
Rental share is taken as the value added of Industry 71 (Renting of machinery and equipment without operator and of personal and household goods) in NACE rev.1.1
dLog(credit/GDP) is the percentage change

Table 2 reports on the regression results from estimating equation (5.2). The key result of the regression is the estimate of the interaction term. However, the magnitude of the parameter needs to be put in perspective. To get a meaningful interpretation, we need to take into account the standard deviation (0.0041)\(^{21}\) of the size of the rental sector across the observed countries by multiplying it with the estimate of the interaction term (−5.295). The resulting estimate suggests that a 1% increase in the size of the rental sector in a country decreases the effect of a negative financial shock by 2.2%. This striking result shows the importance of capital renting in the business cycle and its role in alleviating financial distress.

\(^{21}\)A detailed description of the cross country evidence on the rental sector is available upon request.
6 Conclusion and future work

This paper has provided empirical evidence on the counter-cyclicality of the rental share over the business cycle. We have developed dynamic stochastic general equilibrium model with borrowing constraints and two types of agents, which has performed well in replicating the counter-cyclical behavior of the rental share and matching some of the key business cycle moments observed in the data. The model concentrated on a fundamental trade-off between the two types of capital: only owned capital can serve as collateral, but owning requires more liquidity than renting. Finally, we have shown that the presence of a renting sector alleviates the adverse impacts of a negative financial shock, and these results have been supported by cross-country evidence.

The focus of the present paper has been the aggregate study of the counter-cyclicality of renting, and the implications of firm-heterogeneity are left for future research. Also, the roles of adjustment costs, irreversibility constraints and uncertainty in affecting capital renting may be subjects to further investigations.
7 Appendix

7.1 Data description

See the detailed description of variables below, with the Compustat variable name in parentheses (Source: Compustat manual):

- rental expenses (xrent): This item represents all costs charged to operations for rental, lease, or hire of space and/or equipment.\(^{22}\)

- property, plant and equipment (ppent): This item represents the cost, less accumulated depreciation, of tangible fixed property used in the production of revenue.

- intangible assets (intan): This item consists almost exclusively of the excess of cost over equity acquired in assets of purchased subsidiaries which are still unamortized or not eliminated by a direct charge to a capital account. Examples: copyrights, distribution rights and agreements, organizational expense, franchise and franchise fees, licenses, patents, etc.

- depreciation and amortization (dp): This item represents non-cash charges for obsolescence of and wear and tear on property, allocation of the current portion of capitalized expenditures, and depletion charges.

- amortization of intangibles (am): This item represents a non-cash charge for the systematic write-off of the cost of intangible assets over the period for which there is an economic benefit.

- capital expenditures (capx): This item represents cash outflow or the funds used for additions to the company’s property, plant and equipment, excluding amounts arising from acquisitions, reported in the Statement of Cash Flows.

- average short-term borrowings rate (b astr): This item represents the approximate weighted average interest rate for aggregate short-term borrowings for the reporting year. This item is not available for banks or utility companies.

- employment (emp): This item represents the number of company workers as reported to shareholders. This is reported by some firms as an average number of employees and by some as the number of employees at year-end. No attempt has been made to differentiate between these bases of reporting. If both are given, the year-end figure is used.

\(^{22}\)This item excludes so-called capital leases, which are accounted for among the assets. This is justified on the grounds that capital leases are closer to collateralized lending than to real rentals because ownership is essentially acquired by the lessee. The rental expense measure captures only operating leases, which do not imply a transfer of ownership to the lessee, hence the lessor bears all the risks of ownership. See for more details Table 1 of Eisfeldt and Rampini (2009).
• sales (sales): This item represents an industry segment’s gross sales (the amount of actual billings to customers for regular sales completed during the period) reduced by cash discounts, trade discounts, and returned sales and allowances for which credit is given to customers.

The number of observations, broken down by ISIC-sectors (International Standard Industrial Classification), are shown in Table 3. Industrial and services sectors are equally represented.

Table 3: Number of observations broken down by sectors

<table>
<thead>
<tr>
<th>Industries</th>
<th>Number of firm-year observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td></td>
</tr>
<tr>
<td>Mining and quarrying</td>
<td>10,524</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>100,732</td>
</tr>
<tr>
<td>Electricity, gas and water supply</td>
<td>8,825</td>
</tr>
<tr>
<td>Construction</td>
<td></td>
</tr>
<tr>
<td>Wholesale and retail trade</td>
<td>18,585</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>4,771</td>
</tr>
<tr>
<td>Transport, storage and communications</td>
<td>12,150</td>
</tr>
<tr>
<td>Financial intermediation</td>
<td>54,700</td>
</tr>
<tr>
<td>Real estate, renting and business activities</td>
<td>23,562</td>
</tr>
<tr>
<td>Public administration</td>
<td>746</td>
</tr>
<tr>
<td>Health and social work</td>
<td>3,559</td>
</tr>
<tr>
<td>Other community, social and personal services</td>
<td>5,367</td>
</tr>
<tr>
<td>Unidentified</td>
<td>15,106</td>
</tr>
<tr>
<td>Total</td>
<td>261,149</td>
</tr>
</tbody>
</table>

26
7.2 Benchmark Model

7.2.1 Steady-state

This section provides an analytic description of the steady-state of the benchmark model described in section 3. The steady-state values are denoted by .

The level of owned capital:

$$K^O = \left( \frac{\frac{1}{\gamma} - (1 - \delta^O) - \frac{\lambda}{\gamma} (\beta - \gamma)}{\gamma \alpha \left( \omega^{\frac{1}{\gamma}} + (1 - \omega)^{\frac{1}{\gamma}} \left( \frac{\frac{1}{\gamma} - (1 - \delta^O) - \frac{\lambda}{\gamma} (\beta - \gamma)}{\frac{1}{\gamma} - (1 - \delta^O) - \frac{\lambda}{\gamma} (\beta - \gamma)} \right)^{\frac{\mu-1}{\mu}} \right)} \right)^{\frac{1}{\gamma}}$$  \hspace{1cm} (7.1)

The level of rented capital:

$$K^R = \left( \frac{A}{\alpha E^k \omega^\gamma} \right)^{\frac{\mu-1}{\mu}} (D)^{\frac{1}{\gamma}} \left( \frac{1 - \omega}{\omega} \right)$$  \hspace{1cm} (7.2)

where $A = \frac{1}{\gamma} - (1 - \delta^O) - \frac{\lambda}{\gamma} (\beta - \gamma)$, $D = \frac{\frac{1}{\gamma} - (1 - \delta^O) - \frac{\lambda}{\gamma} (\beta - \gamma)}{\frac{1}{\gamma} - (1 - \delta^O) - \frac{\lambda}{\gamma} (\beta - \gamma)}$, $E = \omega^{\frac{1}{\gamma}} + (1 - \omega)^{\frac{1}{\gamma}} (D)^{\frac{1}{\gamma}} \left( \frac{1 - \omega}{\omega} \right)^{\frac{\mu-1}{\mu}}$, $k = \frac{\mu}{\mu - 1} (\alpha - (\varepsilon - 1)/\varepsilon)$.

The level of composite capital used in production:

$$K = \left[ \omega^{\frac{1}{\gamma}} \left( \frac{A}{\alpha E^k \omega^\gamma} \right)^{\frac{\mu-1}{\mu}} + (1 - \omega)^{\frac{1}{\gamma}} \left( \frac{A}{\alpha E^k \omega^\gamma} \right)^{\frac{\mu-1}{\mu}} (D)^{\frac{1}{\gamma}} \left( \frac{1 - \omega}{\omega} \right)^{\frac{\mu-1}{\mu}} \right]^\frac{1}{\gamma} \hspace{1cm} (7.3)$$

The level of household consumption:

$$C = \left[ \left( \frac{A}{\alpha C^k \omega^\gamma} \right)^{\frac{\mu-1}{\mu}} (\theta \beta + (D)^{\frac{1}{\gamma}} \left( \frac{1 - \omega}{\omega} \right)^{\frac{\mu-1}{\mu}} \right)^\frac{1}{\gamma - 1} \right] \left( \frac{1}{\beta} - 1 \right) \hspace{1cm} (7.4)$$

The level of production:

$$Y = \left[ \omega^{\frac{1}{\gamma}} \left( \frac{A}{\alpha C^k \omega^\gamma} \right)^{\frac{\mu-1}{\mu}} + (1 - \omega)^{\frac{1}{\gamma}} \left( \frac{A}{\alpha C^k \omega^\gamma} \right)^{\frac{\mu-1}{\mu}} (D)^{\frac{1}{\gamma}} \left( \frac{1 - \omega}{\omega} \right)^{\frac{\mu-1}{\mu}} \right]^\frac{1}{\gamma - 1} \hspace{1cm} (7.5)$$

The level of borrowing and saving:

$$B = S = \theta \beta \left( \frac{A}{\alpha E^k \omega^\gamma} \right)^{\frac{\mu-1}{\mu}} \hspace{1cm} (7.6)$$
### 7.2.2 Equilibrium conditions

This section describes the key equilibrium conditions of the model explained in section 3.

\[ \lambda_H^H = \frac{1}{C_t^H} \]  
(7.7)

\[ \frac{1}{R_t} = \beta E_t \left[ \frac{\lambda_{H+1}^H}{\lambda_t^H} \right] \]  
(7.8)

\[ 1 + R_t^R - \delta^R = \beta E_t \left[ \frac{\lambda_{H+1}^H}{\lambda_t^H} \right] \]  
(7.9)

\[ \lambda_E^E = \frac{1}{C_t^E} \]  
(7.10)

\[ \frac{1}{R_t} - \psi_t = \gamma E_t \left[ \frac{\lambda_{E+1}^E}{\lambda_t^E} \right] \]  
(7.11)

\[ 
\gamma E_t \left\{ \frac{\lambda_{E+1}^E}{\lambda_t^E} \left[ \alpha Z_{t+1} K_t^{\alpha-(\varepsilon-1)/\varepsilon} (K_t^R)^{\frac{1}{2}} \right] \right\} = 1 - \gamma E_t \left[ \frac{\lambda_{E+1}^E}{\lambda_t^E} (1 - \delta^O) \right] - \theta \psi_t 
\]  
(7.12)

\[ 
\gamma E_t \left\{ \frac{\lambda_{E+1}^E}{\lambda_t^E} \left[ \alpha Z_{t+1} K_t^{\alpha-(\varepsilon-1)/\varepsilon} (1 - \omega)^{\frac{1}{2}} (K_t^R)^{\frac{1}{2}} \right] \right\} = \gamma E_t \left\{ \frac{\lambda_{E+1}^E}{\lambda_t^E} R_t^R \right\} 
\]  
(7.13)

### 7.2.3 Log-linearised system

This section describes the log-linearized version of the model explained in section 3. Variables written under the sign ` denote percentage deviations from their respective steady-state values.

\[ - \hat{C}_t = \hat{R}_t - \hat{C}_{t+1} \]  
(7.14)

\[ - \hat{C}_t = \beta \left[ \hat{R}^R \hat{R}_t^R - (1 + \hat{R}^R - \delta^R) \hat{C}_{t+1} \right] \]  
(7.15)

\[ \tilde{O} \hat{C}_t + \frac{\bar{S}}{\bar{R}} (\hat{S}_t - \hat{R}_t) + \tilde{K} \hat{K}_t^R \]  
\[ = \tilde{S} \hat{S}_{t-1} + \hat{R}^R \tilde{K}^R (\hat{R}_{t-1}^R + \hat{K}_{t-1}^R) + (1 - \delta^R) \hat{K}^R (\hat{K}_{t-1}^R) \]  
(7.16)
\[ -C_t^e = \bar{C}^e \hat{\psi}_t + \gamma \bar{R}(\hat{R}_t - \hat{C}_{t+1}^e) \]  
(7.17)

\[ \hat{Y}_t = \hat{Z}_t + \alpha \hat{K}_{t-1} \]  
(7.18)

\[ \hat{B}_t = \hat{K}_t^O - \hat{R}_t \]  
(7.19)

\[(\bar{K})^{-\frac{\gamma}{1+\gamma}} \hat{K}_t = \omega^{\frac{1}{2}} (\bar{K}_t^O)^{-\frac{\epsilon-1}{\epsilon}} \hat{K}_t^O + (1 - \omega)^{\frac{1}{2}} (\bar{K}_t^R)^{-\frac{\epsilon-1}{\epsilon}} \hat{K}_t^R \]  
(7.20)

\[ \bar{C}^e \hat{C}_t^e + \bar{B}\hat{B}_{t-1} + \bar{R}^L \hat{K}_t^L + \bar{R}^L \bar{K}_t^L (\hat{R}_t^L + \hat{K}_t^L) \]

\[ = \bar{Y}\hat{Y}_t + (1 - \delta^O) \hat{K}_t^P \hat{K}_{t-1}^P + \frac{\bar{B}}{\bar{R}} (\hat{B}_t - \hat{R}_t) \]  
(7.21)

\[ \gamma \alpha \bar{Z} \bar{K}^{\alpha - \frac{\epsilon - 1}{\epsilon}} (1 - \omega)^{\frac{1}{2}} (\bar{K}_t^R)^{-\frac{1}{2}} [\hat{C}_t^e - \hat{C}_{t+1}^e + \hat{Z}_{t+1} + (\alpha - \frac{\epsilon - 1}{\epsilon}) \hat{K}_t - \frac{1}{\epsilon} \hat{K}_t^R] \]

\[ = \bar{R}^R \bar{R}_t^R \]  
(7.22)

\[ \gamma \alpha \bar{Z} \bar{K}^{\alpha - \frac{\epsilon - 1}{\epsilon}} (\omega)^{\frac{1}{2}} (\bar{K}_t^O)^{-\frac{1}{2}} [\hat{C}_t^e - \hat{C}_{t+1}^e + \hat{Z}_{t+1} + (\alpha - \frac{\epsilon - 1}{\epsilon}) \hat{K}_t - \frac{1}{\epsilon} \hat{K}_t^O] \]

\[ = \frac{\theta}{\bar{R}} \hat{R}_t - \gamma (1 - \delta^O - \theta)(\hat{C}_t^e - \hat{C}_{t+1}^e) \]  
(7.23)

\[ \bar{B}\hat{B}_t = \bar{S}\bar{S}_t \]  
(7.24)

### 7.2.4 Analysis of the non-linear and the log-linearised model

The get a deeper understanding of the cyclical behavior of the model, the non-linear and the log-linearized version of some of the key relationships will be analyzed.

### 7.2.4.1 \( \psi \) and the premium

The shadow price of the borrowing constraint \( \psi_t \) is the difference between the two agents' stochastic discount factors:
\[ \psi_t = D^H_{t,t+1} - D^E_{t,t+1} = \beta \frac{C^H_t}{C^H_{t+1}} - \gamma \frac{C^E_t}{C^E_{t+1}} \]  

(7.25)

Equation (7.25) also suggests that the steady-state level of the shadow price of the entrepreneur’s borrowing constraint is equal to the difference between the two agents’ subjective discount factors:

\[ \bar{\psi} = \beta - \gamma \]  

(7.26)

which suggests that \( \bar{\psi} \) is always positive as long as the entrepreneur is more impatient than the household. The dynamic behavior of \( \psi_t \) around its steady value, \( \bar{\psi} \), in turn depends on the linear combination of the difference between the two agents’ consumption growth rates:

\[ \hat{\psi}_t = \frac{\gamma (\hat{C}^E_{t+1} - \hat{C}^E_t) - \beta (\hat{C}^H_{t+1} - \hat{C}^H_t)}{\beta - \gamma} \]  

(7.27)

The dynamics of \( \psi_t \) has an important effect on the dynamics of the premium as shown in equation (??). The corresponding condition in log-linearized form can be written as:

\[ \frac{\partial \text{premium}_t}{\partial \psi_t} = \frac{1 - \bar{\psi} \theta (1 + \bar{\psi})}{1 - \bar{\psi} (1 + \bar{\psi})} = \bar{\psi} \left( \bar{R} - \theta + \bar{\psi} \bar{R} \hat{R}_t \right) \]  

(7.28)

which implies that under realistic parameter values, a percentage increase in \( \psi_t \) from its steady-state value \( \bar{\psi} \) increases the premium from its steady-state value.

### 7.2.4.2 The marginal products of owned and rented capital

The condition that links the marginal product of owned capital to its implied cost is the following:

\[ 1 = D^E_{t,t+1} MPK^O_{t+1} + D^E_{t,t+1} \gamma (1 - \delta^O) + \psi_t \theta \]  

(7.29)

which implies that the cost of owning one unit of capital, the LHS of (7.29), equals the benefits of owning associated with the three terms of the RHS: (i) the owned capital’s use in tomorrow’s production given by the discounted marginal product of capital, (ii) the discounted future sale price, and (iii) the owned capital’s use as collateral up to \( \theta \) fraction of its value. The explicit version of this condition can be written as:

\[ 1 = \frac{C^E_t}{C^E_{t+1}} \left[ \alpha Z_{t+1} K^\alpha (1 - \delta^O)(1 - \delta^O) + \psi_t \theta \right] \]  

(7.30)

Similarly, the equilibrium condition for renting can be written as:

\[ R^R_{t+1} = \alpha Z_{t+1} K^\alpha (1 + \omega)^{1/2} (K^R_t)^{-1} \]  

(7.31)
The log-linearized version of equations (7.30) and (7.31) can be used to express the dynamics of owned and rented capital around their respective steady-state values:

\[
\hat{K}_t^O = \epsilon \left\{ \hat{z}_{t+1} + \left( \alpha - \frac{\varepsilon - 1}{\varepsilon} \right) \hat{K}_t + \frac{\bar{\psi} \theta \hat{\psi}_t - (1 - \bar{\psi} \theta) \left( \hat{C}_{t+1}^E - \hat{C}_t^E \right)}{1 - \bar{\psi} \theta - \gamma \left( 1 - \delta^O \right)} \right\} \tag{7.32}
\]

\[
\hat{K}_t^R = \epsilon \left\{ \hat{z}_{t+1} + \left( \alpha - \frac{\varepsilon - 1}{\varepsilon} \right) \hat{K}_t - \hat{R}_{t+1}^R \right\} \tag{7.33}
\]

It can be seen that the first two terms in the brackets are common in equations (7.32) and (7.32). Hence, the relative movements of \( \hat{K}_t^O \) and \( \hat{K}_t^R \) entirely depend on the change in the relative magnitudes of the third terms in the brackets. To make these two terms comparable, we do the following: knowing that \( \beta \frac{\hat{C}_t}{\hat{C}_{t+1}} = \frac{1}{\bar{R}} \), and using (7.25), the behavior of \( \hat{R}_{t+1}^R \) is written as:

\[
\hat{R}_{t+1}^R = \frac{\bar{R}}{\bar{R} - 1 + \delta^R} \hat{R}_t = \frac{\gamma \beta}{1 - \beta \left( 1 - \delta^R \right)} \left( \hat{C}_{t+1}^E - \hat{C}_t^E \right) - \frac{\beta - \gamma}{\beta} \hat{\psi}_t \tag{7.34}
\]

The relative movements of \( \hat{K}_t^O \) and \( \hat{K}_t^R \) therefore depends on how the following terms change in relation to each other:

\[
\frac{\left( \beta - \gamma \right) \theta \hat{\psi}_t - (1 - (\beta - \gamma) \theta) \left( \hat{C}_{t+1}^E - \hat{C}_t^E \right)}{1 - (\beta - \gamma) \theta - \gamma \left( 1 - \delta^O \right)} \sim \frac{\beta - \gamma}{\beta} \hat{\psi}_t - \frac{\gamma}{\beta} \left( \hat{C}_{t+1}^E - \hat{C}_t^E \right) \tag{7.35}
\]

Rearranging equation (7.35) and using \( \bar{\psi} = \beta - \gamma \) yields the following, more comparable expression:

\[
\frac{\left( \beta - \gamma \right) \theta \left( \hat{\psi}_t + \hat{C}_{t+1}^E - \hat{C}_t^E \right) - \left( \hat{C}_{t+1}^E - \hat{C}_t^E \right)}{1 - (\beta - \gamma) \theta - \gamma \left( 1 - \delta^O \right)} \sim \frac{\gamma - \beta}{\beta} \left( \hat{\psi}_t + \hat{C}_{t+1}^E - \hat{C}_t^E \right) - \frac{\gamma - 2 \beta}{\beta} \left( \hat{C}_{t+1}^E - \hat{C}_t^E \right) \tag{7.36}
\]

Whenever the RHS of expression (7.36) increases by more than the LHS does, the level of rented capital increases relative to the level of owned capital.

### 7.3 Model with asset prices

#### 7.3.1 Equilibrium conditions

Since most of the equilibrium conditions of the extended model are similar to those obtained for the benchmark model, this section presents those only that are specific to the extended model. Note that the rented capital package, \( K_t^R \), as a control variable only appears in the entrepreneur’s optimization problem, and does not enter the optimization problem of the household. This is because the household controls the components of \( K_t^R \), that is, household capital, \( K_t^H \), and household land, \( L_t^H \). So the demand for rented capital is derived...
directly from the entrepreneur’s optimization problem, whereas the supply of rented capital the Cobb-Douglas production function described by (4.3).

Assuming perfect foresight, the equilibrium conditions are the following:

The household’s demand for land:

$$\beta \frac{C_t}{C_{t+1}} q_{t+1} - \beta \frac{C_t}{C_{t+1}} R^R_{t+1} \frac{MPK^H_{t+1}}{MPK^R_{t+1}} = 0$$

(7.37)

The entrepreneur’s demand for land:

$$\gamma \frac{C^e_t}{C^e_{t+1}} q_{t+1} - \gamma \frac{C^e_t}{C^e_{t+1}} MPL^E_{t+1} + \psi_t q^\star_t \frac{MPK^E_{t+1}}{MPK^O_{t+1}}$$

(7.38)

which has a form similar to equation (4.9). As a result, we have to types of premia: the first one is related to the two types of capital, the other one is related to the two types of land.

The entrepreneurial demand for rented capital:

$$R^R_{t+1} = \alpha Z_{t+1} K^{\alpha-(\varepsilon-1)/\varepsilon}_t (1 + \omega)^{\frac{1}{\varepsilon}} (K^R_t)^{-\frac{1}{\varepsilon}}$$

(7.39)

The entrepreneurial demand of entrepreneurial capital:

$$1 = \gamma \frac{C^E_t}{C^E_{t+1}} \alpha Z_{t+1} K^{\alpha-(\varepsilon-1)/\varepsilon}_t (1 + \omega)^{\frac{1}{\varepsilon}} (K^R_t)^{-\frac{1}{\varepsilon}} + \gamma \frac{C^E_t}{C^E_{t+1}} (1 - \delta^O) + \psi_t q^\star_t \phi K^O_t K^E_t$$

(7.40)

### 7.3.2 Log-linear dynamics

#### 7.3.2.1 Rented capital package, $K^R_t$

The log-linear condition for the demand for rented capital:

$$\hat{K}^R_t = \varepsilon \left\{ \hat{z}_{t+1} + \left( \alpha - \frac{\varepsilon - 1}{\varepsilon} \right) \hat{K}_t - \hat{R}^R_{t+1} \right\}$$

(7.41)

#### 7.3.2.2 Entrepreneurial capital, $K^E_t$

The log-linear condition for the demand for entrepreneurial capital:

$$\hat{K}^E_t = \frac{\varepsilon}{1 - (1 - \delta^O) \gamma} \left\{ \left( 1 - \psi \theta q^* \phi K^O_t K^E_t \right) (\hat{c}^E_t - \hat{c}^E_{t+1}) + \psi \theta q^* \phi K^O_t \left( \hat{v}_t + \hat{q}_t^* + \hat{K}_t^O \right) \right\}$$

(7.42)

which can be written in the following reduced form:

$$\hat{K}^E_t = \Delta_1 (\hat{c}^E_t - \hat{c}^E_{t+1}) + \Delta_2 \left( \hat{v}_t + \hat{q}_t^* + \hat{K}_t^O \right) + \Delta_3 \hat{z}_{t+1} + \Delta_4 \hat{K}_t + \Delta_5 \hat{K}_t^O$$

(7.43)

#### 7.3.2.3 Entrepreneurial land, $L^E_t$

The log-linear condition for the demand for entrepreneurial land:
\[
\hat{L}^E_t = \frac{\varepsilon}{(1-\gamma)q} \left\{ q (\gamma \hat{q}_{t+1} - \hat{q}_t) + (q - \psi \theta q^* (1 - \phi) \frac{K^O_R}{\ell^R} (\hat{c}^E_t - \hat{c}^E_{t+1}) + \psi \theta q^* (1 - \phi) \frac{K^O_R}{\ell^R} (\hat{\psi}_t + \hat{q}^*_t + \hat{K}^O_t) \right\} \\
\quad + \frac{\varepsilon}{(1-\gamma)q} \left[ (1-\gamma) q - \psi \theta q^* (1 - \phi) \frac{K^O_R}{\ell^R} \right] \left[ \hat{z}_{t+1} + (\alpha - \frac{\varepsilon - 1}{\ell}) \hat{K}_t + \frac{\varepsilon - 1}{\ell} \hat{K}^O_t \right]
\]

which can be written in the following reduced form:

\[
\hat{L}^E_t = \Theta_1 (\gamma \hat{q}_{t+1} - \hat{q}_t) + \Theta_2 (\hat{c}^E_t - \hat{c}^E_{t+1}) + \Theta_3 (\hat{\psi}_t + \hat{q}^*_t + \hat{K}^O_t) + \Theta_4 \hat{z}_{t+1} + \Theta_5 \hat{K}_t + \Theta_6 \hat{K}^O_t
\] (7.44)

7.3.2.4 Household capital, \( K^H_t \) The log-linear condition for the household’s capital supply:

\[
\hat{K}^H_t = \frac{(1-\delta^H) \beta + \beta R \theta \phi \frac{K^R}{\ell^R}}{\beta R \theta \phi \frac{K^R}{\ell^R}} (\hat{c}_t - \hat{c}_{t+1}) + \hat{R}^R_{t+1} + \hat{K}^R_t
\] (7.46)

which can be written in the following reduced form:

\[
\hat{K}^H_t = \Lambda_1 (\hat{c}_t - \hat{c}_{t+1}) + \hat{R}^R_{t+1} + \hat{K}^R_t
\] (7.47)

7.3.2.5 Household land, \( L^H_t \) The log-linear condition for the household’s land supply:

\[
\hat{L}^H_t = \frac{q (\beta \hat{q}_{t+1} - \hat{q}_t) + \left[ q \beta + \beta R (1 - \phi) \frac{K^R}{\ell^R} \right] (\hat{c}_t - \hat{c}_{t+1})}{\beta R (1 - \phi) \frac{K^R}{\ell^R}} + \hat{R}^R_{t+1} + \hat{K}^R_t
\] (7.48)

which can be written in the following reduced form:

\[
\hat{L}^H_t = \Phi_1 (\beta \hat{q}_{t+1} - \hat{q}_t) + \Phi_2 (\hat{c}_t - \hat{c}_{t+1}) + \hat{R}^R_{t+1} + \hat{K}^R_t
\] (7.49)
7.4 Parameter-values

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences: Discount factors</td>
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<td>0.99</td>
</tr>
<tr>
<td>Patient household</td>
<td>( \gamma )</td>
<td>0.98</td>
</tr>
<tr>
<td>Impatient entrepreneur</td>
<td>( \gamma )</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Technology parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns to scale parameter</td>
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</tr>
<tr>
<td>Share of renting in composite capital</td>
<td>( \omega )</td>
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<tr>
<td>Elasticity of substitution between the two types of capital</td>
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<tr>
<td>Tightness of the borrowing constraint</td>
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</tr>
<tr>
<td>Persistence of the technology shock</td>
<td>( \rho_Z )</td>
<td>0.91</td>
</tr>
<tr>
<td>Persistence of the financial shock</td>
<td>( \rho_f )</td>
<td>0.91</td>
</tr>
</tbody>
</table>

7.5 Estimation of \( \varepsilon \)

We now describe the method for estimating the elasticity of substitution between rented and owned capital using firm-level dataset from Compustat. We start from the steady state specification, given by equation (??), and convert it, so that it is expressed in terms of observables and parameters which are assumed to be known from other sources. Based on table 4, the annualized parameter values are the following: \( \beta = 0.96 \), \( \gamma = 0.92 \), \( \theta = 0.6 \).

We run the following regression on the cross-section of firms or 2-digit industries:

\[
\log \left( \frac{r_i K^R_i}{K^O_i} \right) = \varepsilon \log \left( \frac{\frac{1}{\gamma - \theta (\beta - \gamma) + \delta^O}}{\delta^R + \delta^R \text{rental fee}} \right) + FE_{industry} + controls_i + u_i, \tag{7.50}
\]

where \( r_i K^R_i / K^O_i \) is the average of the ratio of rental expenses to owned capital, \( i_i \) is the interest rate, \( \delta^O \) is the average depreciation rate, \( \delta^R \) is the depreciation rate for the sectors whose primary purpose is to rent out (average of the depreciation rates in ISIC 70 and 71 sectors). For the firm-level estimation, 2-digit industry fixed effects \( (FE_{industry}) \) are included. Additional controls \( (controls_i) \) include firm age and size. Indices \( i \) refer to either firms or 2-digit industries. The estimation results for 1980-2009 can be seen in Table 5.
Table 5: Estimation results for $\varepsilon$

<table>
<thead>
<tr>
<th></th>
<th>Firm</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>1.221***</td>
<td>1.111*</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.027</td>
<td>0.608</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.573</td>
<td>0.178</td>
</tr>
<tr>
<td>Observations</td>
<td>17539</td>
<td>57</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses.

* and *** denote statistical significance at 10% and 1%.
References


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