Abstract

In a Mortensen-Pissarides model of heterogeneous workers and jobs we investigate the effects of social networks as job information channel on the level of mismatch between workers and firms and we compare the efficiency of the formal market to that of social networks in producing good matches. We assume that employed individuals forward the offers at their disposal to a randomly chosen contact. This behavior might be seen as favoritism: workers recommend each other to any kind of jobs irrespective of productivity. We show that as the probability that ties connect similar agents (homophily) increases, the mismatch level decreases in the society. If this probability is sufficiently high, networks provide good matches at higher rate upon arrival than the formal market, for any efficiency level of the market. In this case the mismatch level is lower in the society with social networks than it would be in a market economy. Hence favoritism can be efficient in our model. Based on the homophily, our model is able to explain the mixed empirical evidence regarding the expected wage comparison between network search and the formal market.

JEL Classification: E24, J62, J64

Keywords: Social Networks, Labor Market Search, Occupational Mismatch, Homophily

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1 Introduction

1.1 Motivation

Workers often use social contacts while searching for a job in addition to formal methods such as newspaper ads or direct application to employers. The different estimates find that 30-70\% of the jobs were obtained through informal methods (see for example, Blau and Robins (1990), Holzer (1987)). Workers being placed by contacts tend to show higher satisfaction with their job (Granovetter (1995)) and experience longer job tenure (Simon and Warner (1992), Loury (2006)). Although this would suggest that worker-employer matches formed through social networks are better on average than those occurring through the markets, estimates of average wages do not fully confirm this idea: Simon and Warner (1992), Kugler (2003), Dustmann et al. (2010) find a wage premium for network search while Bentolila et al. (2010) find a wage discount and Pellizzari (2010) obtains mixed evidence across countries.

In this study, we build an equilibrium search model a la Pissarides (Pissarides (2000)) where we derive the matching function based on assumptions on the transmission of information by social contacts. We consider two types of workers and two types of jobs (two sectors), workers’ productivity is high in one type of job and low in the other. Employers and employees accept bad matches due to market frictions and costly waiting for consecutive matches. Our main questions are the following: what is the impact of connections in the matching process on the mismatch level of the society?; when networks perform better than the market in producing good matches?; what could explain the mentioned mixed evidence regarding wages?

Matching of unemployed workers and vacancies can occur through two channels simultaneously present: the formal market and social networks. We look at the formal market as a random arrival process where we parameterize the efficiency of the market in producing good matches, i.e. the rate at which vacancies reach unemployed workers of high productivity on the market. This parameter could be seen as the intensity of directed search by unemployed and firms toward agents who fit their characteristics. On the other hand, we assume that the arrival to employed agents is not random, but they have direct access to the information about the vacancies of the sector of their current job. Employed workers use the offers at their disposal to pass them to their unemployed social contacts.

Regarding the social networks, first we assume that employed workers transmit offers to a randomly chosen contact of theirs. This practice implies that offers might reach workers who have low productivity on that job, i.e. mismatch is created. The extent to which this happens is determined by the homophily index, i.e. the probability that a link connects two agents of similar
Agents might have good reasons to recommend someone of low productivity for a job despite possibly suffering reputation loss toward the employer. One such reason is that social ties are used in many contexts other than the labor market, for example risk sharing, altruistic reciprocity, and the reputation loss might be compensated by benefits along these dimensions (Beaman and Magruder (2010)). Forwarding job offers might contribute to the maintainance of such beneficial links. Another reason for passing not suitable offers is that such behavior is reciprocated in the future resulting in shorter unemployment spell for the individual. This phenomenon is called favoritism by Bramoullé and Goyal (2009). In our model we will refer to ties willing to favor each other as family.

As an extension, we introduce another type of relationship between the individuals: agents might be endowed with professional acquaintances besides the family contacts. We assume that these acquaintances value less the link connecting them to the individual than they would recommend her to low productivity jobs and loose their reputation toward the employer. Saloner (1985) shows that this reputation effect prevents the job referral from recommending unsuitable workers.\footnote{Beaman and Magruder (2010) show that incentives put by the employer for their employees recommending new workers might be crucial whether the employees refer family members or former co-workers. In our model we do not analyze this game between the referee and the employer, instead we investigate how the aggregate outcomes (mismatch, employment) change when we alter the endowments of the individuals with respect to the different types of connections. In this way our model covers all the possible outcomes of the mentioned game.} There is empirical evidence that work-related contacts (previous co-workers, employers, acquaintances from professional clubs and schools)) provide better paying jobs than family members (Granovetter (1995), Bridges and Villemez (1986)).

Thus in our model the performance of the formal market in providing good offers is set by our efficiency parameter while the performance of the networks is determined by the endowment of the individuals with the different types of social contacts (family and professional) and the homophily level. We investigate how the characteristics of the network influence the mismatch level of the society. Especially, we ask whether favoritism necessarily leads to higher mismatch. We also analyze under what conditions the network is able to produce good matches at higher rate than the market, or vice versa.

Our results show that family ties produce less mismatch if they connect two agents of the same type with higher probability, i.e. the family is more homophilous. An employed family contact of similar type transmits offers of high productivity in the case when she is employed in her sector of high productivity, i.e. the mismatch is low. This follows from the assumption that employed workers hear about vacancies of the sector of their employment and that two agents...
being of similar type have comparative advantage in the same sector. Hence the efficiency of the family channel is connected to the mismatch level of the society. We have an interaction between the presence of family and professional contacts: given that professional contacts always transmit good offers, they increase the probability that the family contact is employed in the good sector, i.e. decrease the level of mismatch. Therefore, the effects of the homophily index are enlarged by the presence of professional contacts. For the same reasons an increase in the market efficiency magnifies the effects of homophily.

Regarding the relative performance of market and social networks, we compare the conditional probability that an offer is of good type upon arrival between the search methods. This probability determines the conditional wage expectation associated to a search method.

First, we focus on the case when every offer is transmitted and the homophily is complete, i.e. family ties always connect similar agents. We find that the family network and the market give the same wage expectation upon arrival, for any level of the market efficiency. The relative probability whether the family contact transmits bad or good offer upon arrival depends on the relative likelihood that she is employed in the bad or good sector. In this case it turns out that this relative likelihood is equal to the probability ratio of bad and good arrivals on the market which is our market efficiency parameter. This happens because the efficiency of the family channel is determined by the mismatch level which is driven by the market efficiency. Hence, we have that the efficiency of both the market and the family is determined by the same factor.

Starting out from this specific case, we can draw two further conclusions. First, given that a decrease in homophily reduces the chance that a good offer comes through family. For incomplete homophily, i.e. when some family ties connect dissimilar workers, the market gives a wage premium over the family network.

Second and more important, if we introduce some other factor which decreases the mismatch, i.e. increases the efficiency of the family, at least in the complete homophily case the family is better than the market. We analyze two such factors: job separation rate difference between good and bad matches and professional contacts. If good matches last longer time than bad ones, in equilibrium there will be more workers employed in good jobs than in bad ones. If there are professional contacts in the network, they create a bias in the network arrival toward good offers which again increases the probability that a family contact is employed in the high productivity sector. We show that in these two cases not only by complete homophily but also by sufficiently high homophily level the family gives a wage premium over the market, no matter how efficient the market is. There is a threshold value of the homophily, by which family and market gives the same wage expectation. This homophily level is increasing in the market efficiency and decreasing in the number of professional contacts and the separation rate difference. In this way, our model is
able to rationalize the empirical findings of Pellizzari (2010) that in some countries social contacts provide a wage premium over the market arrival while in others a wage discount.

We also show that whenever the network search gives a wage premium over the market in terms of conditional expectation, the mismatch level is lower in the economy with social network than it would be when only the market operated as information channel. Hence, whether the presence of networks decreases or increases the mismatch level depends on how homophilous the network is.

Regarding the social network which contains both family and professional contacts, we have that this network provides good matches at higher rate than the market even for lower homophily values. This is again because the professional contacts provide only good offers. However, we find that even if the number of professional contacts is high, some level of homophily in the family relationship is still necessary for the overall network to give a wage premium over the market. If every family tie connects agents of different types, a quite inefficient market already performs better than the overall network. This indicates that the key parameter of our model is the homophily index.

The paper is organized as follows. The next session relates our work to the existing literature. In Section 2 we describe our model, derive the matching function and deduce the equilibrium conditions. Section 3 contains our analytical results while in Section 4 we perform numerical analysis and calibration to the US economy. Section 5 concludes.

1.2 Related literature

Our model is placed in the literature on the role of social networks in job search. A closely related paper is Calvo-Armengol and Zenou (2005) which is built on the Pissarides framework and derives the matching function from assumptions on the information transmission process of the network. They focus on the effect of networks on unemployment rate whereas we investigate the mismatch level in the society. Hence in our model workers and jobs are heterogeneous and we include two types of relationships between agents: family and professional.\(^2\) Calvo-Armengol and Jackson (2007) introduce heterogeneous wages but they do not investigate mismatch since in their model workers are homogenous.

Bentolila et al. (2010) explicitly focus on the effect of networks on mismatch. They assume

\(^2\)Moreover we work in continuous time instead of discrete time which actually takes away the congestion effect producing their main result on non-monotonic relationship between connectivity and unemployment rate.

\(^3\)Other papers which concentrate on unemployment are Boorman (1975), Ioannides and Soetevent (2006), Calvo-Armengol and Jackson (2004).
that the market only transmits good offers while in our model it also provides bad ones. In fact, we parameterize the efficiency of the market in this respect. Further, they assume that contacts have information about different type of jobs at an exogenous rate. In our model the state of the contacts is also endogenous, they might be unemployed or employed in different sectors which implies that they have access to different information. We treat contacts in a symmetric way to the receiver of the information whereas in their model the state of the contacts is exogenous and only the state of the receiver changes endogenously. Hence, we explicitly model the social networks as connections between agents, not only as an extra information channel for a given unemployed. This essentially changes the conclusions about mismatch since we do not obtain that networks always create more mismatch than the market.

The comparison of market and networks with respect to mismatch has mostly been investigated empirically and via comparing average wages of jobs found through formal search methods and social contacts. The most comprehensive paper is Pellizzari (2010) which uses the European Community Household Panel (ECHP) and finds mixed cross-country evidence: for Austria, Belgium and the Netherlands he estimates a wage premium of network search, while for Greece, Italy, Portugal and the United Kingdom he finds a wage discount. Partly using this data Bentolila et al. (2010) obtain wage discount for networks, however they restrict their attention to family contacts. On the other hand, other papers find that networks perform better than market in producing good matches (Simon and Warner (1992), Kugler (2003), Dustmann (2010)). Our model is able to recover both wage premium and discount depending on the level of homophily within the family and the number of professional contacts.

Many papers investigate the effect of job referrals on labor market outcomes (see e.g. Montgomery (1991), Simon and Warner (1992), Tassier and Menczer (2001), Dustman (2010)). They underline the efficiency of social networks in producing good matches which comes from the role of referrals providing additional information compared to the market. Obviously this is a positive effect of networks which is represented by professional contacts in our model. However, we also take into account favoritism (Bramoullé and Goyal (2009)) and show that, for some parameter values, the positive effect of networks on wages still holds in this case as well.

A related literature in sociology compares the quality of job information obtained through strong and weak ties. Granovetter (1995, originally 1974) obtains that work-related contacts provide better job offers than family contacts. This finding has been extended as the hypothesis of "the strength of weak ties" (Granovetter (1973)). This hypothesis says that the connections to acquaintances might be more useful than stronger ties such as family since they are more likely to access information what the individual herself does not have. In our model the advantage of acquaintances is that they provide only high productivity (wage) offers. However, they pass information
less frequently.

There is a growing literature on the role of homophily, the tendency of similar individuals to be connected, in different processes on social networks (Golub and Jackson (2009), Currarini, Jackson and Pin (2009)). Homophily is usually seen as a negative phenomenon since it is connected to segregation of different social groups (Moody (2001)). In particular, Golub and Jackson (2009) show that some type of learning processes might be slowed down in a homophilous network because group boundaries reinforce the heterogeneity of behavior. On the other hand, being connected to similar agents might also be efficient since it also means more effective communication between individuals and easier access to relevant information. In fact, there is evidence that links between similar agents tend to be more resistant to dissolution (McPherson et al. (2001)) indicating that these links are indeed advantageous.

In our model homophily is also positive in the sense that it increases the likelihood that an individual has access to information about offers of the sector where she is more productive in. The dimension of similarity is occupation. We show that the degree of homophily within the family with respect to similarity in occupation is crucial in determining whether social networks or the market provide better paying jobs on average. To our knowledge, we are the first who explicitly focus on the effects of homophily on this question. Regarding the effects of homophily on the labor market outcomes, Van der Leij and Buhai (2008) study the effects of social networks on occupational segregation. They find that homophily in the social networks providing job information implies occupational segregation. This is because workers in the same group choose the same occupation to exploit the efficiency of the job arrival process which rises with homophily.

2 Model

Workers and Occupations. In this model we have two occupations (sectors) $s \in \{A, B\}$ and two types of workers $j \in \{A, B\}$ having $\pi_j$ share in the population. Every worker is able to fill a job of any occupation but $i$ workers have higher productivity in occupation $i$ (good match) than in occupation $j$ (bad match), $i \neq j, i, j \in \{A, B\}$. The productivity in a bad match is $p$ whereas in a good match $\bar{p}$, where $p < \bar{p}$. Unemployed workers earn unemployment benefits $b$ ($b < p < \bar{p}$).

Network structure and information transmission. Every worker has $k$ social contacts which consist of $k_F$ family members (and friends) and $k_P$ professional contacts ($k = k_P + k_F$). For simplicity we analyse the case where $k_F = 1$, i.e. family consists of couples, and $k_P \geq 0$. Later we show that the expected wage of offers obtained through family ties is independent of $k_F$, thus our results for $k_F = 1$ hold in fact for any $k_F$. The contact types refer to the nature of the tie connecting
Figure 1: Network, $k_F = 1$, $k_P = 2$, solid line: family tie, dashed line: professional tie.

two individuals.

The underlying network structure is a regular random graph with degree $k$. The probability that at the two ends of a family link two agents of the same type can be found is $\gamma$, consequently, that the two agents are of different types is $1 - \gamma$. $\gamma$ measures the homophily in the society, i.e. the tendency of individuals of similar characteristics to be connected. Throughout our analysis we focus on the values $\gamma > 0.5$ since this represents the tendency of similar agents being connected.\(^4\)

Professional ties always connect similar agents. What we have in mind here is the idea that these ties are often formed in meetings, schools where similar type agents meet or with the intention of having future access to information of one’s own profession.\(^5\)

Figure 1 illustrates the described network structure. In this graph we have 4 type $B$ and 4 type $A$ agents. Every agent has 2 professional contacts (represented by the dashed lines) and 1 family contact (represented by the solid lines). Professional ties always connect similar type agents, while family ties might connect different ones. The latter happens with probability $1 - \gamma$.

The difference between the contact types is the following: when an offer arrives to a worker who does not need that offer (because she is employed), first she checks with her family member whether he needs the offer (he is unemployed) in which case she passes the offer to him without caring about the type of match to be formed (favoritism). Only if the family tie is employed, she

\(^{4}\gamma = 0.5$ would mean random connections, while $\gamma < 0.5$ would represent a tendency of dissimilar agents to be connected.

\(^{5}$See McPherson et al. (2001) for evidence that individuals of the same occupation tend to be connected and that school and workplace are important places where these connections are formed. There is also wide evidence that family ties are homophilous along many dimensions.
considers the professional contacts as the receiver of the information. In this second step, she only passes the offer if the professional contacts are of the appropriate type and unemployed, hence a good match would be formed. The receiver of the information is randomly chosen among the unemployed professional contacts. If there is no such, the offer is lost (the vacancy stays unfilled). Hence, the information might travel only one step in the network (as it was assumed in Calvo-Armengol and Zenou (2005) and in Calvo-Armengol and Jackson (2004)) which largely facilitates the analysis.

In the following section we derive the matching function and the equilibrium conditions of the model for the case when there is one family contact and the number of professional contacts is kept general. However, we present our results for two general cases: one in which there are no professional contacts and another when we allow for any number of them. In the first case we have a social network where the offers are transmitted to a random neighbor irrespective of her type. This is what we call favoritism. In the second case we add as well the professional contacts but we assume that by the information transmission preference is given to the family contacts. This assumption is consistent with our view that family contacts favor each other and has no influence on our main results regarding the expected wages of the different information channels.

For the derivation of the matching function we use the so-called homogenous mixing assumption which says that the probability that an individual is in some state (unemployed, mismatched or employed in the right sector) is equal to the population frequency of that state. This assumption basically means that the links of the network are randomly drawn at each instant of time (Calvo-Armengol and Zenou (2005)). Hence, for example, the probability that a neighbor of an individual is unemployed is equal to the unemployment rate and is independent of the state of the individual.

**Observability of types.** We assume that when a firm and an unemployed meet they both observe whether their match would constitute a bad or a good one. Thus we suppose that they intentionally accept bad matches. They do this because on a labor market with search frictions they would have to wait too much for a consecutive match. They both incur costs of waiting: for the firm to maintain the vacancy costs $c$, while a worker is better off to earn the wage corresponding to a bad match ($w_B$) than to have the unemployment benefit ($b$).

This situation is more likely to be the case when the productivity of a bad and a good match are close enough. In fact, in a two sector model of heterogeneous workers Moscarini (2001) shows

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6Note that the assumption that family contacts have preference as information receivers further stresses the meaning of favoring relationships. However, neither this assumption nor the restriction that professional ties always connect similar agents are not crucial for our results. The only important assumption is that professional contacts pass only high productivity offers.
that workers whose productivity does not differ much between the two sectors search for and accept offers of both sectors. Thus we focus on the parameter range where the productivity difference is small enough and the firms pay sufficiently high vacancy posting costs.\footnote{If workers or firms did not accept bad matches, we would be back to a one sector model.}

Given this assumption, our professional contacts are not exactly job referrals in the sense that they supply otherwise unknown information. Caring about their reputation they do not introduce bad matches for the employer, they serve as a good channel to contact appropriate workers.

\textbf{Arrival of offers.} Regarding the arrival of offers to individuals, we suppose a technology for each sector which describes the rate of direct arrivals. In a model without social networks this technology would be the matching function itself, however in our model we need to take into account that the offers are passed along the links of the network, hence the matching function is derived from the assumptions on this transmission process.

We assume that there is a difference between the arrival of offers to employed and unemployed agents. We suppose that agents employed in sector $i$ might hear only about the offers of the same sector but not of the other sector. This assumption reflects the idea that employed workers are likely to hear about job openings of their employers or related firms in the same sector. On the other hand, unemployed workers can find any kind of offers when searching on the market, i.e. in our model through direct arrival.\footnote{Observe that these assumptions together with the job information process imply that mismatched workers never recommend a professional contact of theirs to a vacancy since they would have access only to information which does not fit the type of their contact. A professional contact of a given agent provides a job offer only if the contact is employed in the good sector and her family dyad is occupied.} These assumptions imply that the set of direct receivers of the offers of some sector $i \in \{A, B\}$ consists of the following groups: unemployed of type $i$ ($u_i$), unemployed of type $j$ ($u_j$), employed in sector $i$ ($e^i = e^i_i + e^i_j$) which might contain workers of type $i$ ($e^i_i$) and $j$ ($e^i_j$).

We assume that the probability that some offer of sector $i$ reaches someone in the set of candidates is given by:

$$m_i = A v_i^\eta$$

where $v_i$ is the vacancy rate of the corresponding sector, $A$ and $\eta$ are technology parameters with the restriction of $0 < \eta < 1$. The assumption on the value of $\eta$ implies decreasing return for the matches with respect to the vacancy rate: as there are more vacancies posted, the probability that a given vacancy is known by someone decreases ($m_i/v_i = A v_i^{\eta-1}$).

Further, we assume that the unemployed workers of sector $i$ might have higher probability to hear about the offers of this sector than others among the candidates. This is the case when the
market (direct arrival) is efficient enough in matching workers and firms of suitable characteristics or the unemployed workers and firms of a given sector direct their search toward each other.

Using this assumption we write down the probability that an unemployed of type \( i \) receives an offer of sector \( i \), i.e. a good match is formed on the market:

\[
P_{Direct}^G = \frac{\theta u_i}{\theta u_i + u_j + e^i A v_i^{\eta}}
\]

where \( \frac{\theta u_i}{\theta u_i + u_j + e^i} \) is the probability that an unemployed of type \( i \) is chosen as receiver of the information among the candidate agents. The parameter \( \theta \) stands for the mentioned efficiency of the market in producing good matches. If \( \theta = 1 \), the direct arrival is just random. If \( \theta \to \infty \), exclusively the right unemployed group receives the job information.

In the same way, the probability that an unemployed of the other type is reached by a vacancy of sector \( i \) is

\[
P_{Direct}^B = \frac{u_j}{\theta u_i + u_j + e^i A v_i^{\eta}}
\]

The probability that an employed in sector \( i \) hears about an offer of the same sector is

\[
e^i \frac{1}{\theta u_i + u_j + e^i A v_i^{\eta}}.
\]

Note that in this way we exclude the possibility that a mismatched individual improves her position by direct arrival. On the other hand, by the formulation of the transmission of information by the contacts, we have also excluded the possibility that they hear about better offers indirectly. This is because we have assumed that workers having an unneeded offer consider only their unemployed friends as receiver of the information but not the mismatched ones. Thus in this model we disregard the possibility of on-the-job search which simplifies the model in great extend since the job separation will not depend on the network structure but is exogenous.

**Job separation.** Jobs might be destroyed by a productivity shock which makes the productivity of the match so low that the continuation is not beneficial for the parties anymore. We assume that the arrival of such a shock is more probable for bad matches since they have lower productivity than the good matches. Good matches are destroyed according to a Poisson process with parameter \( \lambda \) and the process for bad matches is Poisson as well but with higher parameter: \( \alpha \lambda \) where \( \alpha \geq 1 \). This implies that good matches last longer time than bad ones which is measured by the parameter \( \alpha \).

**Timing.** We work in continuous time which implies that at some instant of time only one event can happen. Thus it cannot happen that an agent is employed and dismissed at the same

\[\text{Note that this formulation also admits an equal separation rate between bad and good matches. We analyze the model for equal and unequal separation rates as well.}\]
instant of time, or that she receives a bad and a good offer at the same time, or that in parallel more than one information source provides offer for her.

2.1 Matching function

The technology defined in the previous section describes the number of "direct encounters" between vacancies and individuals. In our model, however, there is also a possibility to get information through social contacts: unemployed workers might hear about job openings through their employed friends. To construct the probabilities of job finding and vacancy filling we need to consider these two channels, direct and indirect. We define different probabilities for a bad and a good match to be formed. A bad match may occur through direct arrival (market) or family contacts, while for the good match we have an additional channel, the professional contacts.

2.1.1 Bad matches

A bad match occurs when an unemployed worker of type $i$ receives an offer of sector $j$ where $i, j \in \{A, B\}, i \neq j$. According to our assumptions, to obtain such an offer through the network, the information providing contact has to be employed in this sector.

Given the assumption that every individual has only one family member, the probability that an individual of type $i$ in the state of unemployment gets an information through her family is the following:

$$P_{Family}^{B} = u_i \left( \gamma \frac{e_j^i A_i^j}{\theta u_j + u_i + e_j} + (1 - \gamma) \frac{e_j^i A_i^j}{\theta u_j + u_i + e_j} \right)$$

(2)

where $u_i$ is the probability that the receiver of the information is of type $i$ and unemployed. $\gamma \frac{e_j^i A_i^j}{\theta u_j + u_i + e_j}$ is the probability that the contact is of the same $i$ type ($\gamma$) and employed in sector $j$ ($e_j^i$) and is aware of a job opportunity of that sector. The last term stands for the possibility that the family member is of the other type ($j$) and knows a job information.

The probability that a given unemployed agent finds employment in the other sector takes into account the direct and indirect arrival as well. In continuous time these two events cannot happen at the same time, thus we need to sum up the arrival rates of the different channels. So the probability that a bad match is formed is the following:
$$u_i q_B = P_{B}^{Direct} + P_{B}^{Family} = \frac{u_i A^i v_i}{u_i + \theta u_j + e^i} + u_i \left( \gamma \frac{e^j A^j v^j_i}{\theta u_j + u_i + e^j} + (1 - \gamma) \frac{e^j A^j v^j_i}{\theta u_j + u_i + e^j} \right)$$

$$= \frac{u_i A^i v_i}{u_i + \theta u_j + e^i} (1 + \gamma e^j_i + (1 - \gamma) e^j_i)$$ (3)

This latter we may interpret as if $\frac{A^i v_i}{u_i + \theta u_j + e^i}$ was the probability that a vacancy reaches one of the individuals and $u_i (1 + \gamma e^j_i + (1 - \gamma) e^j_i)$ was the probability that this agent is an unemployed of type $i$ or some employed in sector $j$ with a family link to an unemployed of type $i$.

Using this expression, we may write down the bad job filling rate, i.e. the probability that a given vacancy of sector $j$ is filled by a bad match (an unemployed of type $i$):

$$q_B^F = \frac{u_i q_B}{v_j}$$ (4)

### 2.1.2 Good matches

Here we consider the case that an unemployed worker of type $i$ gets an offer of type $i$ ($i \in \{A, B\}$). The information source might be direct arrival, family and professional contacts.

First, we look at the event that the family contact transmits good job information, she might be of type $i$ or type $j$. The probability that an unemployed worker of type $i$ receives job information through her family is given by:

$$P_{G}^{Family} = u_i \left( \gamma \frac{e^j A^j v^j_i}{\theta u_i + u_j + e^j} + (1 - \gamma) \frac{e^j A^j v^j_i}{\theta u_i + u_j + e^j} \right)$$ (5)

The way of constructing this probability is exactly the same as in the case of bad matches. Here the family member has to be employed in sector $i$ in both cases of being of type $i$ or type $j$.

Next, we turn to the channel of professional contacts. They are all of the same type as the unemployed individual herself. According to our assumptions, professional contacts pass information when they are employed in sector $i$ and their family member doesn’t need it (is employed). If these conditions hold, they might choose our given agent among their unemployed professional contacts at random. These latter agents are competitors for our given unemployed worker.

First, we write down the probability that in this competition a given agent receives the information. This formula was derived from the binomial distribution with parameters ($k_P, u$) by Boorman (1975), Calvó-Armengol and Zenou (2005):
Here $s$ is the number of competitors, the probability that $s$ other agents are unemployed apart from our given worker is $u_i'(1 - u_i)^{k_p - s}$ and the probability that among these a given worker gets the information is $1/(1 + s)$.

Second, using this expression we give the probability that an unemployed of type $i$ obtains a good job information through one of her professional contacts:

$$P_{Professional}^G = u_i e_i A^0_i (\gamma (1 - u_i) + (1 - \gamma)(1 - u_i)) 1 - (1 - u_i)^{k_p}$$

(7)

Here $u_i$ is the probability that our given agent is unemployed of type $i$. Her professional contact is always of type $i$ by our assumptions and must be employed in sector $i$ ($e_i^i$) to provide information. $A^0_i$ is the probability that the professional contact is aware of a job information. Upon this event the contact first checks with family member who has to be employed: $(\gamma (1 - u_i) + (1 - \gamma)(1 - u_j))$. This expression take into account that the family member might be of the same or of different type. The remaining part gives the probability that in the competition our given worker receives the information.

Since every agent has $k_p$ professional contacts and at the same time only one of them might have information in continuous time, the probability that some of them provides an offer is $k_p P_{Professional}^G$.

Now we aggregate the different sources of information: market, family or professional contacts. In continuous time only one source can give information at the same time, hence we need to aggregate exclusive events. The probability that a good match is formed is the following:

$$u_i q_G = P_{Direct}^G + P_{Family}^G + k_p P_{Professional}^G =$$

$$\frac{\theta u_i A^0_i}{\theta u_i + u_j + e^i} + u_i \left( \gamma e_i^i A^0_i \left( \frac{e_i^i A^0_i}{\theta u_i + u_j + e^i} + (1 - \gamma) \frac{e_i^i A^0_i}{\theta u_i + u_j + e^i} \right) +
k_p u_i A^0_i \left( \frac{e_i^i A^0_i}{\theta u_i + u_j + e^i} (\gamma (1 - u_i) + (1 - \gamma)(1 - u_j)) 1 - (1 - u_i)^{k_p} \right) \right)$$

$$\left( \frac{\theta u_i}{\theta u_i + u_j + e^i} A^0_i \left( \theta + \gamma e_i^i + (1 - \gamma)e_j^j + k_p e_i^i (\gamma (1 - u_i) + (1 - \gamma)(1 - u_j)) 1 - (1 - u_i)^{k_p} \right) \right)$$

(8)

From here the job filling rate of a vacancy of sector $i$ might be expressed as

$$q_G^F = \frac{u_i q_G}{v_i}$$
where the vacancy is occupied by an employee of high productivity.

### 2.2 Value functions

In this section we write up the value functions associated to the states of the workers and the firms. Workers might be unemployed ($U$) or employed in a bad match ($W_B$) or in a good match ($W_G$). Firms post vacancies which might be vacant ($V$) or filled by a worker of low ($J_B$) or high ($J_G$) productivity.

#### 2.2.1 The worker

The discounted value of unemployment consist of the current value of unemployment benefit and the future value of possible employment which might be in the good or in the bad sector:

$$\delta U = b + q_B(W_B - U) + q_G(W_G - U)$$  \hspace{1cm} (9)

where $\delta$ is the discount rate (common to workers and firms) and $q_B$ and $q_G$ represent the job finding rates of a given unemployed, for a bad and a good match, respectively.

On the one hand, the discounted value of being employed in a bad match is the sum of the value of wages earned ($w_B$) and the possibility of job destruction which happens at a rate $\alpha \lambda$ in case of bad matches.

$$\delta W_B = w_B + \alpha \lambda (U - W_B)$$  \hspace{1cm} (10)

On the other hand, the value function of being in a good match consists of the wages earned ($w_G$) and the future value of being unemployed, here the job destruction rate is $\lambda$:

$$\delta W_G = w_G + \lambda (U - W_G)$$  \hspace{1cm} (11)

#### 2.2.2 The firm

Maintaining a vacancy is costly but it has the future benefits of becoming filled by a worker of either low or high productivity. The value function is given by:

$$\delta V = -c + q_B^F(J_B - V) + q_G^F(J_G - V)$$  \hspace{1cm} (12)

where $c$ is the cost of vacancy and $q_B^F$ and $q_G^F$ are the job filling rates for bad and good matches, respectively.
The value of a bad match comes from the productivity of the worker in a mismatched job $p$ and the future value of job destruction while wages have to be paid for the worker:

$$\delta J_B = p - w_B + \alpha \lambda (V - J_B)$$  \hspace{1cm} (13)

A good match has the same value except that now the productivity of the worker is higher: $\bar{p} > p$ and that a different wage has to be paid:

$$\delta J_G = \bar{p} - w_G + \lambda (V - J_G)$$  \hspace{1cm} (14)

Obviously, the wages paid in a bad match are lower compared to the wage of a good match which compensates the firm for receiving a lower productivity.

Due to the free entry condition for the firms, the competition ensures that the value of the vacancy reduces to zero: $V = 0$. Applying the free-entry condition, we can express $J_B = \frac{p - w_B}{\delta + \lambda}$ and $J_G = \frac{\bar{p} - w_G}{\delta + \lambda}$. Substituting these into the equation (12), we get the job creation curve:

$$c = \frac{q_B F_B}{\delta + \alpha \lambda} + \frac{q_G F_G \bar{p} - w_G}{\delta + \lambda}$$  \hspace{1cm} (15)

### 2.3 Wage determination

Wages are determined by Nash bargaining procedure upon meeting by workers and firms. We assume that the type of the worker and the type of the job is common knowledge between the firm and the worker, hence they condition their decision on the quality of the match.

The wage of a bad match is determined by the following problem:

$$w_B = \text{argmax}_{w_B} (W_B - U)^{\beta} (J_B - V)^{1 - \beta}$$

where $\beta$ is the bargaining power of the worker. This gives the usual first order condition (see the derivation in the Appendix):

$$W_B - U = \beta (J_B + W_B - V - U)$$

In the same way we obtain the FOC for the wage of the good match:

$$W_G - U = \beta (J_G + W_G - V - U)$$

Using the free-entry condition $V = 0$ and the expressions for $J_B$ and $J_G$ we can derive the wages from the FOC’s, see the details in the Appendix.
The expressions for the wages are similar as in the original Pissarides model with the exception that now we have two of them, one for bad and one for good matches:

\[ w_B = \beta p + (1 - \beta)b + \beta c \frac{v}{u} \]  
\[ w_G = \beta \bar{p} + (1 - \beta)b + \beta c \frac{v}{u} \]  

Note that the difference between the two wages is just the productivity difference multiplied by the bargaining power of the worker: \( \beta(\bar{p} - p) \).

2.4 Optimality of decisions

It is optimal for the worker to accept a bad offer now if the discounted value of bad employment is higher than the value of staying unemployed and possibly getting a good offer later. This is:

\[ \delta U_G \leq \delta W_B \iff b + q_G(W_G - U) \leq w_B + \lambda(U - W_B) \iff q_G(W_G - U) - \lambda(U - W_B) \leq w_B - b \]

The last inequality says that the difference between the bad wage and the unemployment benefit should be high enough.

Using the first-order condition of the wage bargaining problem (see Appendix) we can express this condition as a function of the endogenous variables. First, we write:

\[ W_i - U = \frac{\beta}{1 - \beta} J_i = \frac{\beta p_i - w_i}{1 - \beta} \]  

where \( i \in \{B, G\} \).

Second, using this relation we can express the optimality condition of the worker:

\[ \frac{\beta}{1 - \beta} \left( q_G \frac{\bar{p} - w_G}{\delta + \lambda} + \alpha \lambda \frac{p - w_B}{\delta + \alpha \lambda} \right) \leq w_B - b \]

This condition can be evaluated at the equilibrium values of the endogenous variables. We will perform this check by the numerical analysis of the model.

Turning to the firms, we may write their optimality condition in the same way:

\[ \delta V_G \leq \delta J_B \iff -c q_G^F(J_G - V) \leq p - w_B + \alpha \lambda (V - J_B) \iff -c + q_G^F \frac{\bar{p} - w_G}{\delta + \lambda} \leq \frac{p - w_B}{\delta + \alpha \lambda} \]
where the last relation uses the expression for $J_i$ from above and that the value of the vacancies is equal to zero. The condition says that it should have higher value to accept a bad match now than maintaining a vacancy and waiting for a subsequent good match. Again this condition can be evaluated at the equilibrium.

### 2.5 Equilibrium

In this section, we summarize the equilibrium conditions of the model. To be able to give analytical solution, we focus on the symmetric equilibrium. Here the unemployment rate and the rate of being employed in a low or high productivity job are the same between the two groups of workers. In our model this will be the case when the two groups have equal shares in the society ($\pi_A = \pi_B = 0.5$) and are equally homophilous ($\gamma$ is the same for both groups). Moreover, the productivity difference between good and bad matches ($\bar{p} - p$) is the same for the workers of the two types and everybody has the same number of neighbors in the network.

Under these conditions, the vacancies of the two sectors have the same chances to be filled by an employee of low or high productivity and in this way the expected productivity of a vacancy is the same between sectors. This implies that the firms’ incentives to post vacancies are the same in the two sectors, consequently the vacancy rates coincide.

Thus in a symmetric equilibrium the following equalities hold:

- $v_i = v_j \equiv v$
- $u_i = u_j \equiv u$
- $e_i^j = e_j^i \equiv e_B$
- $e_i^j = e_j^i \equiv e_G$

Note that, we have some accounting identities as well: $1 - u_i = e_i^j + e_i^j$ and $1 - u_j = e_j^i + e_j^i$, so in the symmetric case: $1 - u = e_B + e_G$.

Applying these equalities, we write down the set of equations defining the equilibrium. First the job finding probabilities:

$$q_B(u, v, e_B) = \frac{A v^\eta}{1 + \theta u} (1 + \gamma e_B + (1 - \gamma)(1 - u - e_B))$$  \hspace{1cm} (18)

Note that this probability is decreasing in $u$ and increasing in $e_B$ if $\gamma > 0.5$ while decreasing if $\gamma < 0.5$. The effect of the homophily parameter depends on the state of the economy. On the one hand, if the mismatch is high, homophily increases the probability of receiving a bad offer: same
type agents are most probably employed in the bad sector. On the other hand, if the mismatch is low, own type contacts tend to be in good employment, hence they pass information about good matches.

$$q_G(u,v,e_B) = \frac{A v^\eta}{1 + \theta u} \left( \theta + \gamma (1 - u - e_B) + (1 - \gamma) e_B + (1 - u - e_B)(1 - u) \frac{1 - (1 - u)^{k_p}}{u} \right)$$ (19)

The probability of receiving good job information is decreasing in $u$ since unemployed are competitors for the same information. It also decreases in $e_B$ for $\gamma > 0.5$: if there are many agents in bad jobs they will learn mostly about offers of the bad sector hence the probability of receiving a good offer is lower. The effect of $\gamma$ again depends on the state of the economy. Homophily increases the probability to hear about a good offer if there are more people employed in the good sector than in the bad. This probability increases with the number of professional contacts $k_p$ since they exclusively provide good information.

Regarding steady state turnover: in steady state the in- and outflows to employment have to be equal both for good and bad matches. We have the following two turnover equations:

$$\alpha \lambda e_B = u q_B(u,v,e_B)$$ (20)

$$\lambda (1 - u - e_B) = u q_G(u,v,e_B)$$ (21)

The Job Creation equation has been derived in the previous section, this uses the solutions of the wage bargaining problem.

$$c = \frac{u q_B(u,v,e_B) p - w_B}{v} \frac{1}{\delta + \alpha \lambda} + \frac{u q_G(u,v,e_B) \bar{p} - w_G}{v} \frac{1}{\delta + \lambda}$$ (22)

Wages:

$$w_B = \beta \bar{p} + (1 - \beta)b + \beta c \frac{v}{u}$$ (23)

$$w_G = \beta \bar{p} + (1 - \beta)b + \beta c \frac{v}{u}$$ (24)

The equilibrium is defined as follows.

**Definition 1** The equilibrium of the model is a triple $\{u^*, e_B^*, v^*\}$ satisfying the equations (20), (21), (22).

The following proposition states that under some conditions the equilibrium is unique.
Proposition 1 Assume that the parameters of the model are such that:

- \[
A\lambda(\gamma + (1+\gamma)(1+\gamma)\theta + A(1+\gamma)(-1+2\gamma)u\theta)) \bigg/ (A+\theta+At(1-2\gamma)^2) + \alpha \lambda > 0
\]

- \(\lambda \theta > A\)

- \(g(u^*) \equiv (-1+\eta)\lambda(1+\theta u^*) + A(1+P(u^*))u^* < 0\), where \(u^* = 1 - \sqrt{\frac{\theta(1-\eta)}{A(1+k\beta)}}\) and \(P(u) = (1-u)^{1-(1-u)^{\beta\theta}}\)

- either \((-1+\eta)\lambda \theta + A(1+k\beta) < 0\) or if not:

\[
\frac{\partial uq_B(u,v)/v}{\partial u} \bigg|_{u=1} + \frac{\partial uq_G(u,v)/v}{\partial u} \bigg|_{u=1} + \beta cv \left[ \frac{q_B(1,v)}{v(\delta + \alpha \lambda)} + \frac{q_G(1,v)}{v(\delta + \lambda)} \right] = 0
\]

has no solution in \(v\) on the interval \((0,1)\), where

- \(q_B(u,v) = \frac{\alpha \lambda A\nu^\eta [1 + (1-\gamma)(1-u)]}{\alpha \lambda (1+\theta u) - A\nu^\eta u(2\gamma-1)}\)

and

- \(q_G(u,v) = \frac{\lambda A\nu^\eta [\theta + (1-u)(1-\gamma)]}{\lambda (1+\theta u) - uA\nu^\eta u(2\gamma + P(u) - 1)}\)

- \((1+\theta)c > A \left( \frac{(1-\beta)(p-b)}{\delta + \alpha \lambda} + \frac{\theta(1-\beta)(\bar{p} - b)}{\delta + \lambda} \right) - A \left( \frac{\beta c}{\delta + \alpha \lambda} + \frac{\theta \beta c}{\delta + \lambda} \right)\)

Then a unique equilibrium exists.

Proof See Appendix

In the proof of the proposition we follow the usual strategy of showing that the equilibrium is the intersection of the Beveridge curve (turnover equation) having negative slop in the \((u,v)\) space and the Job Creation curve which has positive slop. The difficulty in our model is that the job finding and vacancy filling rates do not have the same properties as in the usual Pissarides model where the matching function is assumed to exhibit some properties which facilitate the analysis. Here first we need to express these probabilities as the functions of \(u\) and \(v\), and not \(e_B\), and after rewrite the conditions such that they depend only on these two variables. Then we give conditions that the Beveridge Curve indeed has positive slop while the Job Creation curve indeed describes a negative relation. Finally, we show that under some condition these curves indeed cross which implies that a unique equilibrium exists.
3 Results

Using the described model, we investigate the impact of the social networks on the mismatch in the society. In particular, we are interested in the effects of the favoring practices and the network structure represented by the number of professional contacts ($k_P$) and the homophily index ($\gamma$). We also compare the performance of the market and the social contacts with respect to producing good matches which contributes to the literature on expected wage comparisons between search methods. We study how this relation is affected by the mentioned network parameters. Later, in the numerical analysis we will also see how these parameters affect the other endogenous variables of the model such as the unemployment rate, vacancy rate and the wages.

We define the quantities of interest of the model. We measure the level of mismatch by the fraction of employed in the bad sector compared to the fraction of employed in the good sector.

**Definition** The level of mismatch is defined as the fraction $e_B/e_G$.

Note that in equilibrium this fraction is equal to $q_B/(\alpha q_G)$, the fraction of arrival rates corrected by the separation rate difference. This follows from the division of the two equilibrium conditions (20) and (21).

For the comparison of the market and social networks with respect to "mismatch creation", we define expected wages of the different search methods. We use the conditional expectation of the wages upon arrival because the different methods have dissimilar arrival rates of offers, thus we need to normalize the expectations by this.

**Definition 2** *The expected wage of the formal search is the conditional expectation of the wage which is obtained through direct arrival conditioned on the event that an offer has arrived:*

$$\frac{\mu Av^\eta}{1+\theta u} W_B + \frac{\mu Av^\eta}{1+\theta u} W_G = \frac{w_B + \theta w_G}{1 + \theta}$$

Hence if the market arrival is random ($\theta = 1$), i.e. "similar" workers and firms do not search for each other, in symmetric equilibrium the direct arrival provides good and bad offers with the same probability and the expected wage is just the average of the wages in good and bad matches. On the other hand, if $\theta > 1$, the expectation puts higher weight on good jobs. Note that in the symmetric equilibrium, $\theta$ means that upon arrival an offer coming through the market is $\theta$ times more likely to be of good type than of bad type.
As for the network channel we define two expectations, one only for the family contacts and another for the family and professional contacts together. Recall that the family contacts forward any offer to each other disregarding the type of the offer.

**Definition 3** The expected wage of the family network is defined as the conditional expectation of the wage obtained through the family member conditional on the event that an offer has arrived through this contact:

\[
\frac{(\gamma e_B + (1 - \gamma) e_G)w_B + (\gamma e_G + (1 - \gamma) e_B)w_G}{e_B + e_G}
\]

Note that the weight on the good wage in this conditional expectation can be re-written as follows:

\[
\frac{\gamma + (1 - \gamma) \frac{e_B}{e_G}}{1 + \frac{e_B}{e_G}}
\]

So the derivative of this weight with respect to \(\gamma\) is

\[
\frac{1 - \frac{e_B}{e_G}}{1 + \frac{e_B}{e_G}}
\]

This derivative clearly increases if the mismatch level \(e_B/e_G\) decreases. Hence in our model, the factors which decrease the mismatch (market efficiency, the number of professional contacts) increase the response of the family wage expectation to the homophily level.

The following Lemma says that as long as employed agents pass the offers to a randomly chosen unemployed neighbor, i.e. they do not direct offers to high productivity contacts on the given job, the wage expectation is independent of the number of (family) contacts. Thus our results for one contact hold for any number of contacts.

**Lemma 1** If family contacts pass the offers to a randomly chosen unemployed neighbor, the conditional expectation of the wage arriving through the family channel is the same for any positive number of family contacts.

**Proof** See Appendix.

This result follows from the fact that upon arrival the average quality of offers received through contacts is independent of the number of contacts which only influences the arrival rate of offers. If the number of neighbors is higher, a given unemployed is more likely to hear of an offer. However, every neighbor has the same chance to be of a given type and be employed in a given
occupation which determines the quality of offers they may transmit. Moreover, the probability that a contact passes an offer to a given unemployed is independent of the type and occupation of the contact. This is because the contact picks some random unemployed in her neighborhood. Hence the average offer quality upon arrival to an unemployed does not change with the number of contacts.

If the family contacts direct the offers, i.e. first they consider the unemployed neighbors who would have high productivity on the given job, we have a "positive" bias in the family arrival as well which implies that the family channel performs even better than it is shown in the following analysis. Therefore, our results give a lower bound on the performance of the family channel.

On the other hand, considering both family and professional contacts, we have the following expectation.

**Definition 4** The expected wage of network search is the conditional expectation of the wage obtained through the network channel (family+professional contacts) conditional on the event that some offer has arrived through this channel:

\[
\frac{Pr_B^N w_B + Pr_G^N w_G}{Pr_B^N + Pr_G^N}
\]

where \(Pr_B^N = \gamma e_B + (1 - \gamma)e_G\) and \(Pr_G^N = (\gamma e_G + (1 - \gamma)e_B) + e_G(1 - u)\frac{1 - u fp}{u} + \alpha q_B\).

Note that we derived these formulas using the arrival rates of the different channels exactly the same way as shown in the section on the derivation of the matching function.

Observe that to compare these expectations it is sufficient to compare the probability weight on the good wages which in the end depends on the ratio of bad and good arrival rates through that channel. For example, in the case of the market, the weight on good matches is just \(\frac{\theta}{1 + \theta}\) which reciprocally depends on the fraction \(1/\theta\). This fraction determines how much would be the mismatch in the society if only the market operated as information channel. In this case we could write:

\[
\frac{e_B}{e_G} = \frac{q_B}{\alpha q_G} = \frac{\alpha \theta}{1 + \alpha \theta} = \frac{1}{\alpha} \frac{1}{\theta}
\]

Here we applied that the measure of mismatch is equal to the ratio of arrival rates corrected by \(\alpha\) which would be equal to the last expression if only the market worked in the society. We can see that it also depends on \(1/\theta\) as the probability weight.

The following Lemma relates the conditional wage expectations of the search methods to the mismatch level of the economy.
Lemma 2 Mismatch level in the society can be written as:

\[
\frac{e_B}{e_G} = \frac{1}{\alpha} \left[ \frac{Pr^M_B}{Pr^M_G} \mu + \frac{Pr^N_B}{Pr^N_G} (1 - \mu) \right]
\]

where \(\mu \in (0, 1)\) and \(Pr^j_i\) is the probability that search method \(j \in \{N(etwork), M(arket)\}\) provides an offer of type \(i \in \{B, G\}\).

**Proof** See Appendix.

We have seen that the conditional wage expectation of a search method depends on the ratio of bad and good arrival rates through that method. This lemma says that the mismatch of the economy can be written as the linear combination of these ratios between the search methods (market and networks). An implication of this lemma is that whenever the expected wage of the network is higher than that of the market, the mismatch level is lower in the economy with social networks than it would be if only the market operated as information channel. In this case the presence of the network improves on the market arrival.

By concentrating on the conditional expectation upon arrival we disregard the differences in the arrival rate of offers between search methods (we normalize the expectations by the arrival rates). Apart from the demonstrated relation between the conditional wage expectation and the mismatch, the focus on this statistics is also justified by the empirical literature on expected wages of the different search methods (Pellizzari (2010), Bentolila et al. (2010)): these papers estimate the same conditional expectation. The estimates are based on the ex-post relation between the earned wages of an employed individual and the search method she used to obtain her actual job. Clearly, this relation conditions on the arrival of offer through that method and the acceptance of the offer.

First, we concentrate on the case where there are no professional contacts (\(k_p = 0\)) and we compare the market with the network of family members. Later we introduce professional contacts, it turns out that there is no qualitative difference between the case when the number of the two types of contacts is equal or there are more professional contacts than family members.

In this section, we derive some analytical results while in the next one we calibrate the model to the US economy.

### 3.1 Model without professional contacts

In the first case, we investigate the effects of the presence of favoring relationships (family) on the mismatch in the economy. We compare the expected quality of the offer coming through the
market and family and study the effects of the homophily ($\gamma$) and market efficiency parameter ($\theta$) on this issue. In the model without professional contacts offers are transmitted to social contacts without regard on the productivity of the resulting match.

Assuming that the market arrival is random ($\theta = 1$), the first proposition says what is the effect of homophily on mismatch when the job separation rate is the same for bad and good matches. The subsequent proposition shows how things change when bad matches have higher separation rate. We also compare the mismatch created by the market to that generated by the family network.

**Proposition 2** Assume that there are no professional contacts ($k_P = 0$) and that the market arrival is uniformly random ($\theta = 1$). If the job separation rate is equal between bad and good matches ($\alpha = 1$):

(i) in equilibrium the fraction of workers employed in bad and good matches are equal ($e_B = e_G$).

(ii) the family provides bad and good matches uniformly random,

(iii) the homophily has no impact on the mismatch and the relative performance of market and family networks.

**Proof** See Appendix.

The intuition behind this result is based on the assumption that employed individuals learn about the offers of their sector of occupation. Obviously, if $\theta = 1$, the direct arrival provides good and bad jobs with equal probability. As for the family network, if the employment rates in good and bad matches are the same, an employed of any type has equal probability to be employed in any kind of job. This implies that the arrival rate of information about bad and good matches through the contacts will be the same. Hence, both sources of information give bad and good offers at uniformly random which implies that the employment rates will be equal ($e_B = e_G$). Homophily has no effect on the equilibrium since own type contacts might be mismatched or in good employment with equal probability.\(^\text{10}\)

Once we introduce the assumption that the job separation rate of the bad matches is higher than that of the good matches, this symmetry of the situation goes away. In this case good matches last longer time implying that in equilibrium there will be more agents employed in good matches

\(^{10}\text{Note that as long as there is no on-the-job search in the society we obtain the same result if the arrival of offers to employed agents is random. In that case employed contacts send any offer they hear about which is equally likely to be bad or good.}\)
than in bad ones. The difference in employment increases with the difference in the separation rates. The consequences of this implication are summarized by the following proposition.

**Proposition 3** Assume that there are no professional contacts \((k_P = 0)\) and that the market arrival is uniformly random \((\theta = 1)\). If the job separation rate of the bad matches is higher than that of good matches \((\alpha > 1)\):

(i) in equilibrium there are more workers employed in good matches than in bad ones \((e_B < e_G)\).

(ii) the mismatch decreases with the homophily index \((\gamma)\) and the separation rate difference \((\alpha)\).
   If \(\alpha\) is higher, \(\gamma\) decreases the mismatch more.

(iii) the family expected wage is higher than the market expectation, the difference is increasing in \(\gamma\).

**Proof** See Appendix.

**Corollary 1** The mismatch level is lower in the economy with social networks than it would be if only the market operated as information source.

**Proof** See Appendix.

If there are more workers employed in high productivity jobs than in low ones, there is higher probability that a family contact is employed in the good sector than in the bad one. Henceforth an unemployed is better off to be connected to a family member of their own type who will provide information about good offers with higher probability than about bad ones. The higher the homophily is, the more likely is that an unemployed receives a good job information which in equilibrium results in lower mismatch. Since at the moment we have assumed that the market arrival is uniformly random, we obtain that for any \(\gamma > 0.5\), the family expected wage will be higher than that of the market. Applying Lemma 2, the fact that the family network provides higher expected wage than the market also implies that the mismatch level in the society is lower in the presence of social networks as information channel.

The next question we treat is what happens if the market arrival is not uniform random, but the market can be efficient at any level \((\theta \in (1, \infty))\).\(^{11}\) In this case there is a positive bias in the market arrival process which implies that the mismatch level decreases, there will be more agents

\(^{11}\)We assume that \(\theta\) is finite, since if it is infinite, every offer directly goes to the unemployed who are productive in that job, and the network has no role in the matching.
in good employment than in bad \((e_G > e_B)\). Now this will be true independently whether the separation rates differ between different types of matches or not. The fact that an employed neighbor has higher probability to be in good occupation than in bad one again causes that the mismatch level decreases with the homophily index. These findings are stated in the next proposition.

**Proposition 4** Without professional contacts \((k_P = 0)\), if the market arrival is biased \((\theta > 1)\),

(i) there are more employed in good jobs than in bad ones \((e_G > e_B)\),

(ii) the mismatch level decreases with \(\theta\) and the homophily index \((\gamma)\). If \(\theta\) is higher, \(\gamma\) decreases the mismatch more.

**Proof** See Appendix.

Turning to the comparison of the different channels with respect to mismatch creation, obviously \(\theta\) increases the performance (expected wage) of the market, while the homophily level that of the family network. However, the market efficiency has a feedback on the performance of the network in providing good offers. Here we use the previously mentioned fact that the effect of the homophily depends on the mismatch level which rises with the market efficiency parameter \(\theta\). An interesting question is what the resulting outcome will be with respect to the relative performance of market and family. The answer depends on whether the job separation rate differs according to the type of the match or not.

**Proposition 5** Assume that there are no professional contacts \((k_P = 0)\) and the market arrival is biased \((\theta > 1)\). If the job separation rate is the same independently of the match type \((\alpha = 1)\):

(i) the expected wage of the market channel is weakly higher than that of the family network,

(ii) homophily \((\gamma)\) increases the expected wage of family, especially, if \(\gamma = 1\), market and family provides good offers at the same rate for any value of \(\theta\).

If bad matches are separated at higher rate \((\alpha > 1)\), there exists a threshold in homophily \(\bar{\gamma}_1\) such that for any \(\gamma \in (\bar{\gamma}_1, 1]\) the family network provides higher expected wage than the market. \(\bar{\gamma}_1\) is decreasing in \(\alpha\).

**Proof** See Appendix.

**Corollary 2** If \(\alpha = 1\) and \(\gamma < 1\) or if \(\alpha > 1\) and \(\gamma < \bar{\gamma}_1\), the mismatch is higher in the economy with social networks than it would be if only the market operated as information source.
Proof See Appendix. 

First, when the job separation rate is similar for every job, the only driving force which reduces the mismatch in the society is the market efficiency. In this case, the direct effect of the efficiency parameter ($\theta$) on the market expectation is higher than the indirect effect which influences the family expectation through the homophily index. As the homophily index increases, the family expectation rises, but the extent of this is never sufficient to make it higher than the market expectation. However, when the homophily is complete ($\gamma = 1$), the two expectations are equal for any value of the market efficiency.\textsuperscript{12,13} In this case the probability that an offer sent by the family contact is of good type is equal to $e_G/e_B$, i.e. the mismatch level in the society, which equal is to $\theta$ in the equilibrium. If we increase $\theta$, the family and market efficiency in producing good matches rises at the same rate.

Second, when the job separation differs between the types of matches, there is an additional factor which reduces the mismatch in equilibrium since good matches last longer time. This fact makes the effect of the homophily index higher and, in contrast to the market efficiency parameter ($\theta$), does not increase the market expectation. The proposition shows that this is sufficient to make the family network to provide higher expected wage than the market for high enough values of the homophily parameter. The threshold in $\gamma$, where the two expectations are equal, is strictly less than one and changes with other parameters of the model, especially with the market efficiency $\theta$ and separation rate difference $\alpha$. The Proposition shows that the threshold decreases with $\alpha$ since a rise in $\alpha$ makes the mismatch lower which implies that the family wage expectation reacts more to the increase of the homophily. So the constant market expectation can be equalized by lower homophily level. We study this kind of comparative statics by the numerical analysis in more detail.

This Proposition also implies that if the good matches last longer and the homophily level is high enough ($\gamma > \tilde{\gamma}_1$), the mismatch level in society with social networks is lower than it would be if only the market channel mediated between jobs and workers. On the contrary, if the homophily is low, the presence of the network makes the matching process less efficient.

In sum, in this section we have studied the effects of social networks on the mismatch level

\textsuperscript{12}Note that complete homophily ($\gamma = 1$) does not mean that contacts only provide good offers. Since with probability $e_B > 0$ they are employed in the bad sector, they have chance to send information about bad jobs.

\textsuperscript{13}Note that Bentolila et al. (2010) has obtained the same result that the network perform strictly worse except for the case of complete homophily. However, they assumed that on the market unemployed exclusively look for good jobs, while in our model this holds for any value of the market efficiency/unemployed search direction parameter $\theta$. We also show that this result is sensitive to other factors such as the assumption on the separation rate equality between match types.
in the society when the social contacts (family members) favor each other and pass any offer to each other independently of the type of the resulting match. We have obtained that the effects of the favoritism depend on the tendency of similar agents to be connected, i.e. the homophily level. Under wide range of the parameters, homophily decreases the mismatch level: the fact that a social contact is of similar type increases the probability of hearing about an offer of good type. Regarding the relative performance of market and family network, we have seen that upon arrival the market provides good offers at weakly higher rate than the family until the job separation rate is the same for both types of matches. Once we introduce that bad matches last less time, the family expectation rises above the market one for high values of the homophily parameter. In this case we also have that the presence of the network channel decreases the mismatch compared to the economy in which only the market operates.

Note that in our model the length of job tenure is connected to the quality of the match which implies that the comparison of average wages between methods indicates the relative length of job tenure for jobs found using those methods. Loury (2006) finds that the jobs found through contacts have a longer tenure than those obtained on the market. Our model is able to reproduce this evidence for the parameter values by which the network gives higher expected wages than the market.

Our results are based on the assumption that employed agents hear about the offers of their actual sector of occupation. We have analysed the case in which this correlation between the type of offer and the sector is perfect, but we conjecture that the same results hold for high enough correlation, when the arrival is sufficiently biased toward the sector of occupation. Finally, as mentioned before, the results do not depend on the assumption that there is only one family contact. Introducing more family contacts would change the absolute value of the arrival rate of bad and good offers but not the ratio of them. This latter quantity is what we analyse by taking the conditional expectation of the wages.

3.2 Model with professional contacts

In the previous section, we have analysed the case when the social networks consist of favoring relationships. However, in reality there also are other type of contacts which are less likely to do favors and they recommend someone for a job only if she will be productive on that job. This type of contacts we call as professional contacts.

Obviously, adding professional contacts to the model creates an additional factor which reduces mismatch in the society since the presence of these contacts rises the arrival rate of good offers. Thus we expect that, similarly to the job separation rate difference parameter ($\alpha$), the num-
ber of professional contacts also rises the effect of the homophily on the arrival of good offers, without influencing the market wage expectation. Hence, in this section we assume that the separation rate is the same in all type of matches ($\alpha = 1$) and focus on the effects of the number of professional contacts. Here we differentiate between two expected wages of the network search: one corresponding to the family channel, the other to the overall network taking into account both family and professional contacts.

First we show that when there are professional contacts among the connections of an individual, in equilibrium there will be more workers employed in good jobs than in bad ones, independently of the value of the market efficiency parameter $\theta$. The level of mismatch decreases with the number of professional contacts since this channel produces only good matches. One implication is that homophily also rises the number of good matches since, as before, contacts of similar type have higher probability to transmit good offers than contacts of dissimilar type.

**Proposition 6** For any positive number of professional contacts ($k_P > 0$), in equilibrium the fraction of workers employed in good jobs is higher than the fraction of employed in bad jobs ($e_G > e_B$). The mismatch ($e_B/e_G$) is decreasing in $k_P$ and in the homophily index $\gamma$. If $k_P$ is higher, $\gamma$ decreases the mismatch more.

**Proof** See Appendix.

This proposition also implies that if the market arrival is uniformly random ($\theta = 1$), the network search performs better than the market for $k_P > 0$ since in this case the only positive bias is attributed to the networks. In particular if $\gamma > 0.5$, the family itself also provides better information than the market. This is completely analogue to the results of Proposition 3 and we omit the proof.

It is more interesting to observe what happens if the market arrival is also biased toward good matches ($\theta > 1$). The following proposition is similar in vein to Proposition 5. It says that for high enough homophily level both the family and the overall social networks provide higher expected wages than the market. The necessary homophily level is lower for the overall network since it includes the positively biased professional contacts as well. Interestingly, for any number of professional contacts, the family network still should be homophilous enough to obtain that the overall network gives a wage premium over the market.

**Proposition 7** If professional contacts are present ($k_P \geq 1$), for any value of the market efficiency parameter $\theta$,
there exists a homophily value $\gamma_1$ such that for any $\gamma \in (\gamma_1, 1]$ the family contact provides higher expected wage than the market,

(ii) $\gamma_1$ is decreasing in the number of professional contacts,

(iii) There exists an interval $(\gamma_2; \gamma_1]$ such that for any $\gamma$ in this interval the market expected wage is higher than the family one but lower than the overall social network’s expected wage,

(iv) if $\theta \geq 2$, $\gamma_2 > 0$,

(v) $\gamma_2$ is decreasing in the number of professional contacts if $\theta \geq 2$.

**Proof** See Appendix.

**Corollary 3** If $\gamma > \gamma_2$, the mismatch is lower in the economy with social networks than it would be if only the market operated as information source.

**Proof** See Appendix.

First of all, we have that for high enough homophily level the family wage is higher than that of the market. Recall the result of the model without professional contacts that for any value of $\theta$ at complete homophily ($\gamma = 1$), the market and family expectations are equal. Here the introduction of professional contacts plays the same role as the difference in job separation rates ($\alpha$): it further decreases the mismatch. This happens in a way such that the market expectation does not rise but the family expectation reacts more to the change of the homophily parameter: it increases more. This implies that for $\gamma$ close to 1, the family expectation gets higher than the market expectation.

Looking at the overall network, it always gives higher expected wage than the family since it includes the professional contacts as well. Hence by the threshold of homophily $\gamma_1$, where the family and market expectations coincide, the overall network still gives a wage premium. Henceforth we can decrease the homophily more to get equality between market and overall network expectations. However interestingly enough, when the market arrival is sufficiently biased ($\theta \geq 2$) and every family link goes to other type agents ($\gamma = 0$), the overall network gives a wage discount compared to the market. This is so despite the presence of any number of professional contacts ($\forall k_p$). Hence we have that the effectiveness of the professional contacts in rising the network expectations is limited and suggests that the homophily level is the key parameter in the determination of the relative performance of networks and market. This is an interesting implication since
in the model there is only one family contact and the number of professional contacts can take any value.

As also shown by the proposition, the importance of professional contacts appears in reducing the required homophily level to have wage premium for the networks. This again goes back to the point that as there are more professional contacts, the mismatch decreases which increases the effect of the homophily level on the overall network and family network expectations. Further comparative statics results of the thresholds are shown in the section of numerical analysis.

The wage premium for the network search also implies that the mismatch level is lower in the economy when the network is present than in the case if only the market mediated between workers and jobs. However, if the homophily is low, the network makes the mismatch higher.

In sum, we have seen that in the model with professional contacts there always exists a homophily level such that the family gives higher expected wages upon arrival than the market. This homophily level is necessarily lower than the complete homophily and this is true for any efficiency level of the market. In this respect the number of professional contacts has the same implications as the job separation difference in the previous section. As for the overall networks, it always gives higher expected wage than the family and also outperforms the market even for lower homophily levels. However, for quite low efficiency levels of the market, when the family networks shows complete heterophily ($\gamma = 0$), the market outperforms the overall network in terms of conditional wage expectations. This means that even though the professional contacts add positive bias to the network arrival, some level of homophily is needed to rise the wage expectation of the overall network above that of the market. However, the presence of the professional contacts decreases the required homophily level. The homophily threshold also determines when social networks make the mismatch lower compared to the level of a market economy.

Based on the findings of this section, our model suggests that wage discount of the network search is most likely to be found in countries where both the homophily level and the number of professional contacts are low. Wage premium is likely in countries where unemployed rely much on professional contacts and the homophily is sufficiently high as well. For intermediate values of homophily a likely situation is that the family contacts give a wage discount while the overall network gives a wage premium over the market. These results might explain the mixed cross-country evidences found by Pellizzari (2010) and also shed some light why using the same dataset Bentolila et al. (2010) find wage discount focusing exclusively on family contacts.
4 Numerical analysis: calibration to the US economy

In this section we calibrate the model for the US economy at a quarterly frequency and analyse how high the homophily threshold should be for the network to give a wage premium over the market. We are interested how the homophily threshold is influenced by the other parameters of the model: market efficiency ($\theta$), job separation difference ($\alpha$) and number of professional contacts ($k_P$). We will investigate what is the effect of the network parameters ($\gamma$ and $k_P$) on other endogenous variables of the model such as the unemployment and vacancy rates and wages.

4.1 Calibration

Table 1 summarizes the parameter values and the reasons for the choices. The bargaining power of the workers $\beta$ is set to 0.5, a value mostly used in the literature (Mortensen and Pissarides (1999), Fontaine (2008)). The discount rate $\delta$ is set to 0.988 which corresponds to a quarterly interest rate of $r = 0.012$ used by Shimer (2005). As for the quarterly separation rate $\lambda$, we use the value of 0.1 as it is estimated by Shimer (2005). In our model this is the separation rate of the good matches and we assume that the separation rate of bad matches is double, $\alpha = 2$. We have experimented with other values of this parameter as well, as discussed later in this section (see Figure 2). The productivity in a good match $\bar{p}$ is normalized to 1. We assume that the productivity in a bad match is 15% less than the productivity in a good match, we set $p = 0.85$. Note that to be optimal to accept a bad match by the firms and the workers, we need to keep the productivity of the two match types to be close enough to each other. We will see that by this value of $p$ both firms and workers act optimally in the equilibrium.

The elasticity of the matching function with respect to the vacancy rate ($\eta$) is set to be 0.3 which is the lower bound of the estimates summarized by Petrongolo and Pissarides (2001). We choose the lower bound to satisfy the condition of unique equilibrium stated in Proposition 1. For the same reason we set the market efficiency parameter $\theta$ to 30 but we experiment with other values later.

The two network parameters, the number of professional contacts $k_P$ and the homophily index $\gamma$ are the parameters of our interest and we vary them on a grid: $\gamma \in [0.6, 1]$, $k_P \in [1, 25]$. However, for the calibration we use two benchmark values: $k_P = 10$, $\gamma = 0.7$.

The remaining three parameters, the efficiency parameter of the matching function ($A$), the cost of vacancy posting ($c$) and the value of the unemployment benefit are calibrated to match three real world statistics of the US economy: the unemployment rate being 5.67%, the market tightness ($v/u$) being 0.634 (both of them estimated by Shimer (2005)) and the unemployment
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reason</th>
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<tr>
<td>$\beta$</td>
<td>0.5</td>
<td>Standard in the literature</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>Benchmark, varied later</td>
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<tr>
<td>$\delta$</td>
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<td>Quarterly interest rate $r = 0.012$, Shimer (2005)</td>
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<tr>
<td>$b$</td>
<td>0.473</td>
<td>Calibration*</td>
</tr>
<tr>
<td>$\bar{p}$</td>
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<td>Normalization</td>
</tr>
<tr>
<td>$p$</td>
<td>0.85</td>
<td>Assumption</td>
</tr>
<tr>
<td>$c$</td>
<td>0.36</td>
<td>Calibration*</td>
</tr>
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<td>Estimated by Shimer (2005)</td>
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<td>Benchmark, varied later</td>
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<td>Petrongolo and Pissarides (2001)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>30</td>
<td>To satisfy conditions of Proposition 1</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>Assumption</td>
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Table 1: Parameter values. Calibration is carried out to match the following statistics: unemployment rate: 0.0567 (Shimer (2005)), market tightness: 0.634 (Shimer (2005), unemployment benefit replacement rate: 0.5567 (OECD statistics).

benefit replacement rate\(^{14}\) being 55.67\% in 2007 (see the OECD labor market statistics). These statistics are matched by the following values: $A = 0.32$, $c = 0.36$, $b = 0.473$.

Note that by these parameter choices the conditions to guarantee a unique equilibrium are satisfied and both firms and workers make optimal decisions to accept bad matches instead of waiting for a good match.

4.2 Results

In this section we solve the model numerically using the calibrated parameter values. First, we will see how large the homophily threshold values are which separate the different cases with respect to the relative performance of networks and market. Second, we study the effects of the network parameters, the number of professional contacts and homophily level, on the other endogenous variables of the model, such as unemployment rate, vacancy rate and wages.

First, we illustrate the results obtained in the case when professional contacts are not present

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\(^{14}\)The replacement rate is the average unemployment benefit/wage ratio. In our model this is unemployment benefit $b$ divided by the average wage $\frac{w_B + w_G}{2}$.
Figure 2: Threshold levels of the homophily parameter for different values of $\theta$ and $\alpha$ when there are no professional contacts ($k_p = 0$).

in the network. Here Proposition 5 stated that for any value of the market efficiency ($\theta$), if the good matches last longer, there always exist a threshold of the homophily index such that above this level the family wage expectation is higher than that of the market. This threshold is shown on Figure 2 for different values of $\theta$ and the job separation difference rate $\alpha$.

We can see that when the separation rate is equal across all kind of matches ($\alpha = 1$), the homophily threshold is always 1. This result was shown in the first part of Proposition 5, saying that in this case at complete homophily ($\gamma = 1$) market and family always give the same expected wages. As we increase the value of $\alpha$, the mismatch decreases in the society which makes the effects of the homophily stronger: the derivative of the family wage expectation with respect to the homophily level rises. For high enough homophily the market expectation is already lower than the family one. This situation is reached at lower homophily level if $\alpha$ is higher or the market is
Figure 3: Threshold levels of the homophily parameter for different values of $\theta$ and $k_P$ when separation rates are the same ($\alpha = 1$). Left panel: $\theta = 15$, right panel: $\theta = 30$

not so efficient, i.e. the value of $\theta$ is small.\textsuperscript{15} Thus the threshold value of the homophily index is increasing in the market efficiency and decreasing in the job separation rate difference.

Second, on Figure 3 we illustrate the homophily thresholds described in Proposition 7 for the case when professional contacts are present in the model and there is no difference in the separation rates ($\alpha = 1$). Recall that in this case we have two thresholds, one by which the family expected wage is equal to the market expectation and an other by which the overall network expectation takes the same value as the market expected wage.

On the left panel we show the threshold values for $\theta = 15$. There are three areas separated by the two mentioned threshold values of the homophily parameter. When homophily is low,\textsuperscript{15} Note that for example $\theta = 5$ means that upon arrival, an offer coming through the market is five times more likely to be of good type than of bad type.
the market expectation is the largest. For intermediate values of homophily, the overall network, which includes the channel of professional contacts, gives a wage premium over the market but the family channel alone still gives a wage discount. We also can see a tiny region of quite high homophily values where the family contacts also provide a wage premium over the market. We also can see that these threshold values are decreasing in the number of professional contacts since more professional contacts mean less mismatch in the model which implies that the homophily increases more the expected wage of the network channels.

On the right panel, we show the same regions but now for a higher value of market efficiency: $\theta = 30$. We can see that in this case the region where the market performs the best is larger while the other two regions are smaller. This is because as the market is more efficient, it’s expected wages are higher and the network should be more homophilous to equalize this higher expectation. Note that this is the case despite the fact that with the rise of market efficiency the mismatch is again smaller in the society which implies that the homophily can increase the network expectation more. However, this effect is not sufficient to make the homophily threshold to be decreasing in $\theta$. In the Appendix, we also show the same figure for the case when the separation rate for bad matches is double compared to the rate for good matches. In this case, the thresholds are lower for the reasons described earlier (see Figure 6).

On Figure 4 we plot the difference in expected wages between the market and the different network channels. Here we apply the parameter setting described in the section of calibration (i.e. $\theta = 30$, $\alpha = 2$). On left panel the difference between the family and the market expectation is plotted as the function of the number of professional contacts for different values of the homophily level. We can see that the number of professional contacts has no effect on this difference and that the family channel gives a wage discount except for the highest value of the homophily ($\gamma = 1$).\textsuperscript{16} We also can see that the wage difference is quite small for any of these parameter values (a suitable comparison is the wage panel of Figure ?? where we can see that the wages are around 0.85).

The right panel of Figure 4 graphs the expected wage difference between the overall network and the market. Obviously, here the network performs better since it also includes the professional contacts who always provide offers of high wage. We can see that, in accordance with the previous graphs, the network performs better and better as we increase the number of professional contacts for any value of the homophily parameter. This is because the necessary homophily threshold decreases with the number of professional contacts. However, for high homophily levels the network always gives a wage premium over the market, independently of the value of $k_P$. Note that even though it seems on the figure that for sufficiently many professional contacts the network

\textsuperscript{16}Note that by this parameter setting the homophily threshold is strictly less than 1 and it is between 0.95 and 1.
always gives a wage premium, this is not the case in general, especially if we increase the value of market efficiency. However, it is true that for any $\theta$ by complete homophily ($\gamma = 1$) the network always gives a wage premium. This illustrates the importance of the homophily parameter.

We also investigate how the two network parameters, $\gamma$ and $k_P$ affect the other endogenous variables of the model, like unemployment and vacancy rates and wages (see Figure 5). On the first panel, we can see that the unemployment rate is getting lower as there are more professional contacts. Obviously, the matching process is more effective if the network is more connected. If we increase of the homophily parameter, the unemployment rate should decrease. This is because in this case the mismatch decreases, which leads to lower average separation rate. The respective results are shown in Table 2 in the Appendix 7.3. However, as we see on Figure 5, this effect is not
too strong, at least when we assume that good matches last twice as long on average as bad ones ($\alpha = 2$).

Direct consequence of the previous analysis is that the fraction of individuals employed in low productivity jobs is decreasing with homophily and the number of work-related contacts (see the third and forth panel of the figure). The two last panels show that the wages increase with $k_P$ but are unaffected by the homophily index. This is because as the unemployment rate gets lower due to the more connected network, the outside option of the workers improves which makes their bargaining power higher. Hence they are able to claim higher wages.

As for the vacancy rate (second panel), if the unemployment reduces, the job filling rate of a given vacancy decreases as well, which puts lower incentives for vacancy posting. The wage increase has the same effect. Hence, with the increase of the number of work-related contacts the firms post less vacancies. The homophily again does not have significant effect but in small it has the same effects as the number of professional contacts.

5 Conclusions

We have investigated the question what the impact of social networks on mismatch of workers to occupations is, whether networks perform better or worse than the market in this respect. In our model individuals have two types of relationships: family contacts who favor each other and pass any offer even though it does not fit the abilities of the receiver, and professional contacts who forward only good offers caring about their reputation.

We have shown that if the probability that the family ties connect agents of similar type (homophily index) increases, the level of mismatch in the society decreases. This effect is larger if there are other factors in the society which decrease mismatch such as the presence of professional contacts or the fact that good matches last longer time than bad ones. If unemployed can use more professional contacts, the mismatch level obviously decreases.

Regarding the relative performance of market and networks we have differentiated between the family network and the overall network which contains both types of relationships. We have shown that whether market performs better in producing good matches compared to any of these two networks is determined by the homophily level of the family network.

If there are professional contacts in the network or there is difference in the job separation rate between bad and good matches, for sufficiently high level of homophily the family provides a wage premium over the market. This is true for any efficiency level of the market. The necessary homophily level is increasing with the market efficiency and decreasing with the number of profes-
sional contacts and the extent of separation rate difference. We have also shown that whenever the network gives a wage premium, the mismatch in the society is lower than it would be if only the market operated as information channel. Hence favoritism is not necessarily bad from an efficiency point of view, the presence of social networks leads to higher productivity.

Interestingly, we have obtained that we need a certain homophily level even for the overall network to give a wage premium over the market despite that it might contain many professional contacts who always send good offers. If every family tie connects dissimilar agents, a relatively inefficient market already provides a wage premium over the overall network. However, we need less homophily for the overall network to give wage premium over the market than for the family network. This explains why we find wage discount for the family ties and wage premium for the overall networks in some countries.

6 References

- Beaman, L. and Magruder, J. (2010): Who gets the job referral? Evidence from a social network experiment, Manuscript


• Dustmann, Ch., Glitz, A. and Schönberg, U. (2010): Referral-based job search networks, Manuscript


• Loury, LD (2006): Some contacts are more equal than others: Informal networks, job tenure and wages, Journal of Labor Economics,


7 Appendix

7.1 Calculation of wages

The problem to be solved is the following:

\[ w_l = \arg\max_{w_l} (W_l - U)^\beta (J_l - U)^{1 - \beta} \]

where \( l \in \{B, G\} \)

FOC:

\[ (J_l - V)^{1-\beta} \beta (W_l - U)^{\beta-1} \frac{\partial (W_l - U)}{\partial w_l} + (1 - \beta) (J_l - V)^{-\beta} \frac{\partial (J_l - V)}{\partial w_l} (W_l - U)^{\beta} = 0 \]

Similarly to the literature, we assume that \( \frac{\partial U}{\partial w_l} = 0 \), by the free-entry condition \( V = 0 \).

Further we have \( \frac{\partial W_l}{\partial w_l} = 1 \) and \( \frac{\partial J_l}{\partial w_l} = -1 \). This leads to the following simplified version of the FOC:

\[ \beta J_l = (1 - \beta) (W_l - U) \]

Now we can substitute the expressions of the value functions. In the text we have seen that:

\[ J_l = \frac{p_l - w_l}{\delta + \lambda_l} \]

where \( p_l \) is the corresponding productivity depending on the type of the match and \( \lambda_l \) is just \( \lambda \) for good matches while it is \( \alpha \lambda \) for bad matches.

Using the expressions for the value of employment for the worker (10) and (11), we get:

\[ W_l = \frac{w_l + \lambda_l U}{\delta + \lambda_l} \]

Substituting into the FOC we can express the wages as the function of \( U \) and the parameters:

\[ w_l = \delta U + \beta (p_l - \delta U) \]

What is left is to express \( \delta U \) as the function of the parameters. In the text we had:

\[ \delta U = b + q_B (W_B - U) + q_G (W_G - U) \]

Using the FOC we substitute out \( (W_l - U) \) and we get:

\[ \delta U = b + \frac{\beta}{1 - \beta} (q_B J_B + q_G J_G) \]
Using (22) and the relationship between the job filling and job finding probability \((q^F_i = \frac{uq}{v})\), we have the following:

\[
(q_BJ_B + q_GJ_G) = c\frac{v}{u}
\]

Hence we get the following for \(\delta U\):

\[
\delta U = b + \frac{\beta}{1-\beta}c\frac{v}{u}
\]

Using this latter and substituting into the expression for wages:

\[
w_l = \beta p_l + (1-\beta)b + \beta c\frac{v}{u}
\]

which is similar to the standard wage equation in the basic Pissarides model.

### 7.2 Proofs

#### 7.2.1 Lemma 1

**Proof** The conditional expected wage of the family channel when the family consists of couples \((k_F = 1)\):

\[
\frac{(\gamma e_B + (1-\gamma) e_G)w_B + (\gamma e_G + (1-\gamma) e_B)w_G}{e_B + e_G}
\]

Under the assumption of the Lemma, the probability that a given unemployed obtains a low productivity job through her family contacts for general \(k_F\) is given by the following:

\[
q^F_B = k_F \frac{Av^\eta}{1+\theta u} \left((1-\gamma)e_G + \gamma e_B\right) \sum_{s=0}^{k_F-1} \left(\frac{k_F - 1}{s}\right) u^s (1-u)^{k_F-1-s} \frac{1}{s+1} =
\]

\[
k_F \frac{Av^\eta}{1+\theta u} ((1-\gamma)e_G + \gamma e_B) \frac{1 - (1-u)^{k_F}}{uk_F} \quad (25)
\]

Agents might receive offers from \(k_F\) family contacts. Any of these neighbors have an offer with probability \(\frac{Av^\eta}{1+\theta u}\). The offer can be of low productivity with probability \(((1-\gamma)e_G + \gamma e_B)\), i.e. it has to come either from an agent of different type employed in their corresponding good sector or from an agent of the same type employed in their bad sector. The last term stands for the probability that our given unemployed is chosen as the receiver of the information at random among the other candidates. This latter expressions follows the same logic of binomial random draws as in the case of professional contacts. As we can see, this expression depends only on the unemployment rate which follows from the assumption that the sender of the information does not distinguish between different types of unemployed.
Similarly, we write down the probability that a given unemployed obtains a good offer through one of her family contacts:

\[ q_F^G = k_F \frac{Av_G}{1 + \theta u} ( (1 - \gamma)e_B + \gamma e_G) \frac{1 - (1 - u)^k_F}{uk_F} \]  

(26)

Observe that the two expressions, \( q_F^G \) and \( q_B^G \), differ only in the terms in the middle: \( (1 - \gamma)e_B + \gamma e_G) \) and \( (1 - \gamma)e_G + \gamma e_B) \).

Given these arrival rates we can write down the conditional expectation of the family wage upon arrival:

\[ \frac{q_F^G w_B + q_B^G w_G}{q_F^G + q_B^G} = \frac{((1 - \gamma)e_G + \gamma e_B)w_B + ((1 - \gamma)e_B + \gamma e_G)w_G}{e_G + e_B} \]

We get this equality because all the common terms between \( q_B^G \) and \( q_G^G \) fall out. Thus, the conditional expectations of the wage in the case of \( k_F > 1 \) and \( k_F = 1 \) are the same.

7.2.2 Lemma 2

We define \( Pr_j^i \) as the probability that method \( j \in \{M, N\} \) (standing for ’Market’ and ’Networks’) provides an offer of type \( i \in \{B, G\} \). As we have indicated above in the text, when we compare conditional expected wages of the search methods, in fact, we compare the ratios \( Pr_G^i / Pr_B^i \). This ratio is what would determine the mismatch if only method \( j \) operated in the society as information channel.

Here we show that the mismatch level of the society can be written as the linear combination of these ratios across the search methods. This fact implies that whenever the network performs better than the market in terms of expected wages, the mismatch is lower than it would be in the case if only the market provided information about the vacancies.

The mismatch level is given by the following expression:

\[
\frac{e_B}{e_G} = \frac{q_b}{aq_G} = \frac{1}{\alpha} \left[ \frac{Pr_B^M + Pr_N^B}{\frac{Pr_G^M + Pr_N^B}{Pr_G^M + Pr_N^B}} \right] = \frac{1}{\alpha} \left[ \frac{Pr_B^M}{Pr_G^M} + \frac{Pr_N^B}{Pr_N^B} \right] = \frac{1}{\alpha} \left[ \frac{Pr_B^M}{Pr_G^M} + \frac{Pr_N^B}{Pr_N^B} \right] \left[ Pr_G^M + Pr_N^B \right] = \frac{1}{\alpha} \left[ \frac{Pr_B^M}{Pr_G^M} + \frac{Pr_N^B}{Pr_N^B} \right] \left[ (1 - \mu) \right]
\]

where \( \mu \in (0, 1) \). The first equality uses the equilibrium relations (20) and (21), the second equality uses that the good and bad arrival rates are sums of good and bad arrival rates by search methods (market and networks), respectively.

If the network provides higher expected wage upon arrival than the market, \( Pr_B^F / Pr_G^F < Pr_B^M / Pr_G^M \), implying that the mismatch, as the linear combination of these two ratios, is lower than the higher of the two \( (Pr_B^M / Pr_G^M) \).
7.2.3 Proposition 1

Assume that the parameters of the model are such that:

- \( \frac{\alpha(\lambda)(1 + \gamma)(1 + \gamma + \eta) - A(1 + \alpha)(1 + \gamma) - \alpha \theta}{(1 + \theta + A(1 - 2\gamma))} + \alpha \lambda > 0 \)

- \( \lambda \theta > A \)

- \( g(u^*) \equiv (-1 + \eta) \lambda(1 + \theta u^*) + A(1 + P(u^*)u^* < 0, \text{ where } u^* = 1 - \sqrt[\delta + \lambda + \beta c]{\frac{q^\theta(1, v) + q^\theta(1, v)}{v(\delta + \alpha \lambda)}} }\)

- either \((-1 + \eta) \lambda \theta + A(1 + k_p) \leq 0 \) or if not:

\[
\frac{\partial q_B(u, v)}{\partial u} + \frac{\partial q_G(u, v)}{\partial v} + \beta c v = 0
\]

has no solution in \( v \) on the interval \((0, 1)\). where

\[
q_B(u, v) = \frac{\alpha \lambda Av^\eta[1 + (1 - \gamma)(1 - u)]}{\lambda(1 + \theta u) - Av^\eta u(2\gamma - 1)}
\]

and

\[
q_G(u, v) = \frac{\lambda Av^\eta[\theta + u - \gamma(1 - u)]}{\lambda(1 + \theta u) - Av^\eta u(2\gamma + P(u) - 1)}
\]

\[
(1 + \theta)c > A \left( \frac{(1 - \beta)(p - b)}{\delta + \alpha \lambda} + \frac{\theta(1 - \beta)(\bar{p} - b)}{\delta + \lambda} \right) - A \left( \frac{\beta c}{\delta + \alpha \lambda} + \frac{\theta \beta c}{\delta + \lambda} \right)
\]

Then a unique equilibrium exists.

**Proof Uniqueness** For the proof of uniqueness we need to show that the Beveridge curve is strictly decreasing and the job creation curve is strictly increasing in the \((u, v)\) plane. For these, first we need to express the job finding probabilities as the functions of \( u \) and \( v \). We can use (20) to express \( e_B = \frac{q_B}{\alpha \lambda} \). Substituting this identity into (18), we can find \( q_B \) as a function of \( u \) and \( v \):

\[
q_B(u, v) = \frac{\alpha \lambda Av^\eta[1 + (1 - \gamma)(1 - u)]}{\alpha \lambda(1 + \theta u) - Av^\eta u(2\gamma - 1)}
\]

We may do the same with respect to \( q_B \), now using (21), we have \( e_B = 1 - u - \frac{q_B}{\alpha \lambda} \). We write (19) in the following way:

\[
q_G(u, v, e_B) = \frac{Av^\eta}{1 + \theta u}[\theta(1 - u)(1 - \gamma) - e_B(1 - 2\gamma - P)]
\]

46
where \( P(u) = (1 - u)^{1-(1-u)\gamma} \) and \( P = 0 \) if \( k_P = 0 \), \( P > 0 \) when \( k_P > 0 \). Some properties of this function are summarized after this proof.

Again, substituting \( e_B \) we get \( q_G(u, v) \):

\[
q_G(u, v) = \frac{A\eta\lambda^2[\theta + (1 - u)(1 - \gamma)]}{\lambda(1 + \theta u) - uA\eta^2(2\gamma + P - 1)}
\]  

(29)

To get rid of \( e_B \) from the equilibrium conditions as well, we use \( e_B = \frac{q_{uu}}{a\lambda} \), and substitute it into (21). Hence, the equation for the Beveridge curve in the \((u, v)\) plane is:

\[
a\eta q_{G}(u, v) + uq_{B}(u, v) - \alpha\lambda(1 - u) = 0
\]

Now, we show that this is negatively slopped under some conditions. First, we may see that the left-hand side increases in \( v \). The derivative of \( q_B(u, v) \) with respect to \( v \):

\[
\frac{\partial q_B(u, v)}{\partial v} = \frac{A\eta\lambda^2(2 + \gamma(-1 + u) - u)(1 + \theta u)v^{-1+\eta}}{(\lambda + \lambda\theta u - A(-1 + 2\gamma)uv^\eta)^2} > 0
\]

The sign of this expression depends on the sign of \((2 + \gamma(-1 + u) - u) = 1 - \gamma + 1 + u(\gamma - 1) > 0\) since even though \( \gamma - 1 < 0 \) its absolute value is less than 1.

On the other hand, the derivative of \( q_G(u, v) \):

\[
\frac{\partial q_G(u, v)}{\partial v} = \frac{A\eta\lambda^2(1 + \theta + \gamma(-1 + u) - u)(1 + \theta u)v^{-1+\eta}}{(\lambda + \lambda\theta u - A(-1 + 2\gamma + P)uv^\eta)^2}
\]

which is positive in the same way since \( \theta \geq 1 \).

Second, we see that the left-hand side is increasing in \( u \) under some conditions. To see this, first, we give condition such that the derivative of the right-hand side is positive at \( u = 1 \). The derivative of the lhs is:

\[
A\lambda v^\eta \left( \frac{\lambda(2 + \gamma(-1 + u) - u)}{(\lambda + \lambda\theta u - A(-1 + 2\gamma)uv^\eta)^2} + \frac{\alpha\lambda(1 + \theta + \gamma(-1 + u) - u)}{(\lambda + \lambda\theta u - A(-1 + 2\gamma + P)uv^\eta)^2} \right) + \\
A\lambda v^\eta \left( (-1 + \gamma)u \left( \frac{1}{\lambda + \lambda\theta u - A(-1 + 2\gamma)uv^\eta} + \frac{\alpha}{\lambda + \lambda\theta u - A(-1 + 2\gamma + P)uv^\eta} \right) \right) + \\
A\lambda v^\eta \left( \frac{\alpha\lambda(1 + \theta + \gamma(-1 + u) - u)u^2v^\eta P_{\alpha}}{(\lambda + \lambda\theta u - A(-1 + 2\gamma + P)uv^\eta)^2} \right) + \alpha\lambda
\]

(30)

We evaluate this derivative at \( u = 1 \). Note that \( P(1) = 0 \), and \( P_\alpha(1) = -1 \). We obtain the following:

\[
A\lambda v^\eta \frac{(\lambda + \theta + \gamma(1 + \gamma)) - \gamma(1 + 2\gamma)}{(\lambda + \lambda\theta + Av^\eta(1 - 2\gamma))} + \alpha\lambda
\]

47
Now we provide a condition such that this derivative is positive. Notice that $\gamma + (-1 + \gamma)\theta + \alpha(-1 + \gamma + \gamma\theta) = (\gamma - 1)(\theta + \alpha) + \gamma(1 + \theta\alpha) > 0$ since $\gamma > 1 - \gamma$ and $1 + \theta\alpha \geq \theta + \alpha$ if $\alpha \geq 1$ and $\theta \geq 1$. Hence the derivative can be negative only if the positive term $Av^\eta((1 + \alpha)(-1 + \gamma)(-1 + 2\gamma) + \alpha\theta)$ is high enough. This is the highest at $v = 1$. Moreover, at $v = 1$ we multiply this possibly negative number with the highest value of $A\lambda v^\eta$ and divide it with the lowest value of the denominator.

Hence, the best chance to be negative is at $v = 1$. The following condition says that the derivative has to be positive at this value as well:

$$\frac{A\lambda (\lambda(y + (-1 + \gamma)\theta + \alpha(-1 + \gamma + \gamma\theta)) - A((1 + \alpha)(-1 + \gamma)(-1 + 2\gamma) + \alpha\theta))}{(\lambda + \lambda\theta + A(1 - 2\gamma))^2} + \alpha\lambda > 0 \quad (31)$$

Now we show that if the derivative is positive at $u = 1$ then it is positive for any value of $u < 1$. To this end we need to show that the Beveridge curve is concave in $u$.

First we see that $uq_B(u, v)$ is concave:

$$\frac{\partial(uq_B(u, v))}{\partial^2 u} = -\frac{2A\lambda^2 v^\eta (-\lambda(-1 + \gamma + (-2 + \gamma)\theta) + A(-2 + \gamma)(-1 + 2\gamma)v^\eta)}{(\lambda + \lambda\theta u - A(-1 + 2\gamma)uv^\eta)^3}$$

Here the denominator is positive which is implied by the fact that the denominator of $q_B(u, v)$ has to be positive. This derivative is negative if $-\lambda(-1 + \gamma + (-2 + \gamma)\theta) + A(-2 + \gamma)(-1 + 2\gamma)v^\eta > 0$. Again, this latter term has to be positive in the worst case when the second negative term is the highest in absolute value: at $v = 1$. We can simplify the condition to:

$$\lambda(1 - \gamma) + (2 - \gamma)(\lambda\theta - A(2\gamma - 1)) > 0$$

A sufficient condition is $\lambda\theta > A$ (which comes from evaluating the condition at $\gamma = 1$ where the value of the left-hand side is the smallest).

Next, we show that under the same condition, $f(u) \equiv uq_G(u, v)$ is concave in $u$ as well. We use the following definition of concavity:

$$f(tx + (1 - t)y) \geq tf(x) + (1 - t)f(y)$$

where $t \in [0, 1]$. In our case the function has to be concave for any $u \in [0, 1]$, hence $x = 0$ and $y = 1$. Note that $f(0) = 0$. This implies the following condition:

$$\frac{(1 - t)\lambda(\theta + (1 - \gamma)t)}{\lambda(1 + t\theta) + AV^\eta(1 - 2\gamma - P(t))} \geq \frac{(1 - t)\lambda\theta}{\lambda(1 + \theta) + A\nu^\eta(1 - 2\gamma)}$$

First, we show that the nominator of the left-hand side is higher than that of the right-hand side:

$$(1 - t)\lambda(\theta + (1 - \gamma)t) \geq (1 - t)\lambda\theta \iff t(1 - \gamma)(1 - t) \geq 0$$
This is true since \( t \in [0, 1] \) and \( \gamma \leq 1 \).

Second, we show that the denominator of the left-hand side is smaller than that of the right-hand side when \( \lambda \theta > A \):

\[
\lambda(1 + t\theta) + A\nu^\theta(1 - 2\gamma - P(t)) < \lambda(1 + \theta) + A\nu^\theta(1 - 2\gamma)
\]

This might be rearranged in the following way:

\[
(t - 1)(\lambda \theta + A\nu^\theta(1 - 2\gamma)) - A\nu^\theta P(t) < 0
\]

A sufficient condition for this is \( \lambda \theta > A\nu^\theta(2\gamma - 1) \) which is the most stringent if \( v = 1 \) and \( \gamma = 1 \): \( \lambda \theta > A \).

This implies that if \( \lambda \theta > A \), \( uq_G(u, v) \) is strictly concave. This implies that if \( uq_G(u, v) \) was increasing in \( u \) at the point \( u = 1 \), then it is increasing on the whole interval \([0,1]\).

Putting everything together, we have seen that \( auq_G(u, v) + uq_B(u, v) - \alpha A(1 - u) \) is increasing in \( v \) and under some conditions it is decreasing in \( u \). These properties imply that the Beveridge curve is decreasing in the \((u, v)\) plane. Take any point in the curve, increase \( v \), \( auq_G(u, v) + uq_B(u, v) - \alpha A(1 - u) \) increases. To get back to the equilibrium, we need to decrease \( u \).

As a second step we need to show that the Job Creation (JC) curve is increasing in the \((u, v)\) plane. We state here the equation of the JC curve again:

\[
c = \frac{uq_B(u, v, e_B)}{v} \tilde{p} - w_B + \frac{uq_G(u, v, e_B)}{v} \tilde{p} - w_G
\]

where \( w_B = \beta p + (1 - \beta)b + \beta c_u \) and \( w_G = \beta \tilde{p} + (1 - \beta)b + \beta c_u \) which are decreasing in \( u \) and increasing in \( v \).

First, we show that under some conditions the right-hand side is decreasing in \( v \). Sufficient conditions are \( uq_B(u, v)/v \) and \( uq_G(u, v)/v \) being decreasing in \( v \):

\[
\frac{\partial uq_G(u, v)}{\partial v} = A\alpha(1 + \theta)\lambda(1 + \theta)(\gamma(-1 + u) - u)v^{-2+\eta} (\alpha(-1 + \eta)\lambda(1 + \theta)u + A(-1 + 2\gamma)uv^\gamma) \]

\[
\frac{\partial uq_G(u, v)}{\partial v} = (\alpha + \lambda \theta)u - A(-1 + 2\gamma)uv^\gamma)^2
\]

This derivative is negative only if \( h(u) \equiv \alpha(-1 + \eta)\lambda(1 + \theta)u + A(-1 + 2\gamma)uv^\gamma < 0 \) since \( (2 + \gamma(-1 + u) - u)>0 \) as shown before. This condition can be re-written: \( \alpha(-1 + \eta)\lambda + u(\alpha(-1 + \eta)\lambda + A\nu^\theta(2\gamma - 1)) < 0 \). A sufficient condition for this to hold is: \( \alpha(1 - \eta)\lambda \theta > A \).

Now we turn to the derivative of \( uq_G(u, v)/v \) with respect to \( v \):

\[
\frac{\partial uq_G(u, v)}{\partial v} = A\lambda(1 + \theta + \gamma(-1 + u) - u)v^{-2+\eta} ((-1 + \eta)\lambda(1 + \theta)u + A(-1 + 2\gamma + P)uv^\gamma) \]

\[
\frac{\partial uq_G(u, v)}{\partial v} = (\lambda + \lambda \theta u - A(-1 + 2\gamma + P)uv^\gamma)^2
\]
This is negative only if \((-1 + \eta)\lambda(1 + \theta u) + A(-1 + 2\gamma + P)uv^\gamma < 0\) where the second term is positive and is the highest when \(v = 1\) and \(\gamma = 1\). Substituting these values we get \(g(u) \equiv (-1 + \eta)\lambda(1 + \theta u) + Au(1 + P(u))\). We show that \(g(u)\) has a maximum on \([0, 1]\) and we require the function to be negative even at this maximum.

Some properties of \(g(u)\): \(g(0) = (\eta - 1)\lambda < 0\), \(g(1) = (\theta + 1)\lambda(\eta - 1) + A\), \(g'(u) = (-1 + \eta)\lambda\theta + A(1 + k)(1 - u)^k\), \(g'(1) = (-1 + \eta)\lambda\theta < 0\), \(g'(0) = (-1 + \eta)\lambda\theta + A(1 + k)\), \(g''(u) = -Ak(1 + k)(1 - u)^{-1+k} < 0\).

Hence, we have that \(g(u)\) is convex. If \(g'(0) < 0\), \(g(u)\) takes it’s maximum at \(u = 0\) where the value of the function is negative, hence it is negative on the whole interval between 0 and 1.

On the other hand, if \(g'(0) > 0\), the function has an interior maximum on \([0,1]\) since \(g'(1) < 0\) and the function is strictly convex on \([0,1]\). The maximum is characterized by \(g'(u) = 0\) and given by:

\[
u^* = 1 - \frac{\lambda \theta (1 - \eta)}{A(1 + k)}\]

We require \(g(u)\) to be negative at it’s maximum: \(g(u^*) < 0\). Note that this condition also implies the one which makes \(uq_B(u,v)/v\) to be decreasing in \(v\), since \(h(u) \leq g(u) < 0\) for any \(u\). This is true because \(a \geq 1\) and \(P(u) \geq 0\).

We also need to show that the right-hand side of the JC curve is increasing in \(u\). The derivative of the right-hand side with respect to \(u\) is given by:

\[
\frac{\partial q_B(u, v)}{\partial u} \frac{p - w_B}{\delta + \alpha \lambda} + \frac{\partial q_G(u, v)}{\partial u} \frac{\bar{v} - w_G}{\delta + \lambda} + \frac{\beta c}{u^2} \left[ \frac{uq_B(u, v)}{v(\delta + \alpha \lambda)} + \frac{uq_G(u, v)}{v(\delta + \lambda)} \right]
\]

where \(\beta c \frac{p}{u^2}\) is the derivative of \(p - w_B\) and \(\bar{v} - w_G\) with respect to \(u\).

We would like to give a condition on the parameters which guarantees that this this derivative is positive. The problem is that the derivative of \(uq_B(u,v)\) and \(uq_B(u,v)\) might be negative for large enough values of \(u\). Since these functions are concave, as we have shown it before, the derivatives are the smallest at \(u = 1\). If they are positive at this point, the proof is done. If they are negative, we need to evaluate the whole expression at \(u = 1\). Note that \(p - w_B\) and \(p - w_G\) are the highest at \(u = 1\) hence the negative derivatives get the highest multiplicator at this point. Moreover, the last term in the derivative, which is always positive, is the lowest at \(u = 1\) if the derivative of the first two terms are negative here. All this implies that if the whole derivative of the Job Creation curve is positive at \(u = 1\), it is positive for any \(u\).

To give a condition on the parameters we need to get rid of the other endogenous variable \(v\) as well. Here we have the problem that the derivative is a complicated function of \(v\): neither decreasing, nor increasing and neither convex, nor concave on the whole range \([0,1]\). To solve this
problem, first, we show that if \( v \to 0 \), the derivative goes to infinity. Second, we state the required condition in the form that for the appropriate parameters, the derivative as a function of \( v \) cannot have a root on the interval \((0,1]\) which implies that it is strictly positive on the whole interval.

We write out the derivative in parts evaluated at \( u = 1 \):

\[
\frac{\partial uq_B(u,v) / v - w_B}{\partial u} = \frac{\lambda v^\gamma \beta c}{\alpha \lambda (1 + \theta) - A(-1 + 2 \gamma) v^\gamma} +
\]

\[
A\alpha \lambda v^\gamma \beta c (\alpha \lambda (1 + \theta) - A(-1 + 2 \gamma) v^\gamma) +
\]

\[
A\alpha \lambda v^{\gamma-1} (\alpha \lambda (1 + \theta) - A(-1 + 2 \gamma) v^\gamma) (b(-1 + \beta + p - \beta(p - p + cv))) / (\alpha \lambda (1 + \theta) - A(-1 + 2 \gamma) v^\gamma)^2
\]

(32)

Here if we take the limit \( v \to 0 \), the denominator goes to the positive finite number \( \alpha \lambda (1 + \theta) \). The first term of the nominator goes to 0. The second term of the nominator might be re-written as follows:

\[
A\alpha \lambda v^{\gamma-1} \alpha \lambda (1 + \theta) + A\alpha \lambda v^\gamma \beta c + A\alpha \lambda v^\gamma \\
- A\alpha \lambda v^{\gamma-1} (\alpha \lambda (1 + \theta) - A(-1 + 2 \gamma) v^\gamma) (b(-1 + \beta + p - \beta(p + cv))) (\alpha \lambda (1 + \theta) - A(-1 + 2 \gamma) v^\gamma)^2
\]

(33)

This is a polynomial of \( v \), if \( v \) goes to 0, those terms where the power of \( v \) is positive go to 0 and those where it is negative go to infinity. Given that \( \eta < 1 \) we have at least one term of the latter type. Hence, the whole expression goes to infinity if \( v \) goes to 0.

We can do the same reasoning on the remaining part of the derivative of the Job Creation curve.

\[
\frac{\partial uq_C(u,v) / v - w_C}{\partial u} = A\lambda v^{\gamma} \beta c \theta \quad \text{and} \quad \frac{\lambda (1 + \theta) - A(-1 + 2 \gamma) v^\gamma}{A\lambda v^{\gamma-1} \lambda(1 + \gamma + \gamma \theta) - A\lambda v^\gamma (-1 + \gamma)(-1 + 2 \gamma - 2 \theta)} \]

(34)

Here again the first term goes to 0 as \( v \) approaches 0. The second term’s denominator goes to a finite positive value \( \lambda (1 + \theta) \). The nominator can be re-written as follows:

\[
A\lambda v^{\gamma-1} \lambda(-1 + \gamma + \gamma \theta)(b(-1 + \beta + p - \beta(p - p + cv))) + A\lambda v^\gamma \beta c A\lambda v^\gamma \lambda(-1 + \gamma + \gamma \theta)
\]

(35)
The second and forth term of this expression go to 0 when \( v \) goes to 0 while the first term goes to infinity. The third might go to zero or to infinity depending on the concrete value of \( \eta \). Hence the whole expression goes to infinity.

What we have seen is that the derivative of the Job Creation curve goes to infinity as \( v \) approaches to zero. This is important for us since it says that for some values on the interval \( v \in [0, 1] \) the derivative should be positive. We would like that for any \( v \) in this interval it would be positive which is the case when the derivative is never zero on this interval. Hence, the parameters of the model should be such that the derivative is never zero which is possible to verify for any concrete value of the parameters:

\[
\frac{\partial uq_B(u, v)}{\partial u} \frac{p - w_B}{\delta + \alpha \lambda} + \frac{\partial uq_G(u, v)}{\partial u} \frac{\bar{p} - w_G}{\delta + \lambda} + \frac{\beta c v}{u^2} \left[ \frac{uq_B(u, v)}{v(\delta + \alpha \lambda)} + \frac{uq_G(u, v)}{v(\delta + \lambda)} \right] \neq 0
\]

at \( u = 1 \).

Under the mention conditions if we increase \( u \) from a given point in the \((u, v)\) plane, the right-hand side of the Job Creation curve increases. To get back to equilibrium we need to increase \( v \) as well, since in this case the right-hand side decreases. The Job Creation curve is positively slopped in the \((u, v)\) plane.

Therefore, BC describes a monotone negative relationship in \((u, v)\) while JC describes a positive one. Hence, they might cross only once, the equilibrium is unique if exists.

**Existence.** We show that under some condition the Beveridge and Job Creation curves actually cross. First, we demonstrate that at \( u = 0 \) the Beveridge curve lies above the Job Creation curve. In fact the vacancy rate \( v \) solving the equation of the Beveridge curve goes to infinity as \( u \) tends to 0. To see this we write the Beveridge curve in the following way:

\[
\alpha q_B(u, v) + q_B(u, v) - \frac{\alpha \lambda (1 - u)}{u} = 0
\]

If we take the limit \( u \to 0 \), the last term of the left-hand side goes to infinity, while \( q_B(u, v) \to Av^\eta(2 - \gamma) \) and \( q_G(u, v) \to Av^\eta(1 - \gamma + \theta) \). Hence, the equation can hold only if these last two terms go to infinity at the same rate as the last term of the left-hand side. To this we need that \( v \) goes to infinity.

Turning to the Job Creation curve, as \( u \) tends to zero, the vacancy rate \( v \) solving the equation of the JC curve has to go to zero as well. This becomes clear writing the JC the following way:

\[
cv = \frac{uq_B(u, v, e_B)(p - w_B)}{\delta + \alpha \lambda} + \frac{uq_G(u, v, e_B)(\bar{p} - w_G)}{\delta + \lambda}
\]

If \( u \to 0 \), the expressions on the right-hand side go to 0 as well, hence the left-hand side has to go to zero which is the case when \( v \to 0 \).
Second, we show that at $u = 1$, the vacancy rate solving the Beveridge Curve is 0 while $v$ solving the Job Creation curve is positive and finite. If $u = 1$, only the direct arrival might provide offers, evaluating the arrival rates defined in equations (18) and (19) at $u = 1$, $e_B = 0$, we obtain:

$$q_B(1, v, 0) = \frac{A v}{1 + \theta}$$

$$q_G(1, v, 0) = \frac{\theta A v}{1 + \theta}$$

Looking at the Beveridge Curve evaluated at $u = 1$:

$$\alpha q_G(1, v) + q_B(1, v) = \alpha \frac{\theta A v}{1 + \theta} + \frac{A v}{1 + \theta} = 0$$

This equation can hold only if $v = 0$.

As for the Job Creation curve, it can written at $u = 1$ as follows:

$$(1 + \theta)c = A v^{\eta - 1} \left( \frac{(1 - \beta)(p - b)}{\delta + \alpha \lambda} + \frac{\theta(1 - \beta)(\bar{p} - b)}{\delta + \lambda} \right) - A v^{\eta} \left( \frac{\beta c}{\delta + \alpha \lambda} + \frac{\beta c}{\delta + \lambda} \right)$$

If $v \to 0$, the expression on the right-hand side goes to infinity: the last term goes to zero, while the first tends to infinity since $\eta < 1$. Hence, $v = 0$ cannot be a solution. The right-hand side is decreasing in $v$, so as we increase $v$ from 0, it decreases. If we assume that

$$(1 + \theta)c > A \left( \frac{(1 - \beta)(p - b)}{\delta + \alpha \lambda} + \frac{\theta(1 - \beta)(\bar{p} - b)}{\delta + \lambda} \right) - A \left( \frac{\beta c}{\delta + \alpha \lambda} + \frac{\beta c}{\delta + \lambda} \right)$$

$v = 1$ cannot be a solution either since at this point the right-hand side is already smaller than the left-hand side. By the monotonicity and continuity of the right-hand side in $v$, this implies that there should be a solution of the Job Creation curve in the interval (0,1).

In sum, we have seen that the Beveridge Curve is above the Job Creation curve at the point $u = 0$ while the reverse is the case at $u = 1$. Hence, by the continuity of both curves, there should be an intersection of the two for some value of $u$ in the interval (0,1).

Lemma 3 Some properties of $P(u)$:

1. $P(u)$ is decreasing in $u$,
2. $P(0) = 1$, $P(1) = 0$,
3. $P_u(1) = -1$, $P_u(0) = -0.5k(1 + k)$.
Proof 1. $P(u)$ can be rewritten as:

$$P(u) = (1 - u)[1 + (1 - u) + \cdots + (1 - u)^{k_{P-1}}]$$

which is clearly decreasing in $u$.

3. 

$$P_u \equiv \frac{\partial P}{\partial u} = \frac{-1 + (1 - u)^k (1 + ku)}{u^2}$$

which is clearly -1 at $u = 1$.

$$\lim_{u \to 0} P_u = \lim_{u \to 0} \frac{-k(1 + k)(1 - u)^{-1+k}u}{2u} = \lim_{u \to 0} \frac{k(1 + k)(1 - u)^{-2+k}(-1 + ku)}{2} = \frac{-k(1 + k)}{2}$$

7.2.4 Proposition 2

Proof Proposition 2 assumes the following parameter values: $k_P = 0$, $\theta = 1$, $\alpha = 1$. In this case the offer arrival probabilities are given by the following equations (see the expressions in (18) and (19)):

$$q_B(u, v, e_B, e_G) = \frac{Av^\gamma}{1 + u}(1 + \gamma e_B + (1 - \gamma)e_G)$$

$$q_G(u, v, e_B, e_G) = \frac{Av^\gamma}{1 + u}(1 + \gamma e_G + (1 - \gamma)e_B)$$

By dividing the two equilibrium conditions (20) and (21) with each other, we get that:

$$\frac{q_B}{q_G} = \frac{1 + \gamma e_B + (1 - \gamma)e_G}{1 + \gamma e_G + (1 - \gamma)e_B} = \frac{e_B}{e_G}$$

Rearranging this latter equality we obtain:

$$e_B(1 + \gamma e_G + (1 - \gamma)e_B) = e_G(1 + \gamma e_B + (1 - \gamma)e_G)$$

$$e_G - e_B + (1 - \gamma)(e_G^2 - e_B^2) = 0$$

$$(e_G - e_B)(1 + (1 - \gamma)(e_G + e_B)) = 0$$

since the second term is always positive, the multiplication can be zero only if $e_G = e_B$.

If $\theta = 1$, the market provides bad and good offers with probability 0.5 each. Substituting $e_B = e_G$ into the definition (3) implies that the family arrival puts equal weights on the two wages as well which are independent of the homophily index $\gamma$. By $e_B = e_G$, $q_B$ and $q_G$ do not depend on $\gamma$ either. $$\blacksquare$$
7.2.5 Proposition 3

Proof (i) Proposition 3 assumes the following parameter values: $k_P = 0$, $\theta = 1$, $\alpha > 1$. The arrival rates of bad and good offers are the same as in the previous proof:

$$q_B(u, v, e_B, e_G) = \frac{A\nu}{1 + u} (1 + \gamma e_B + (1 - \gamma)e_G)$$

$$q_G(u, v, e_B, e_G) = \frac{A\nu}{1 + u} (1 + \gamma e_G + (1 - \gamma)e_B)$$

However, the equilibrium condition changes now:

$$\frac{q_B}{\alpha q_G} = \frac{1 + \gamma e_B + (1 - \gamma)e_G}{\alpha (1 + \gamma e_G + (1 - \gamma)e_B)} = \frac{e_B}{e_G}$$

Rearranging this latter equality we obtain:

$$\alpha e_B (1 + \gamma e_G + (1 - \gamma)e_B) = e_G (1 + \gamma e_B + (1 - \gamma)e_G)$$

$$e_G - \alpha e_B + (1 - \gamma)(e_G^2 - \alpha e_B^2) + e_G e_B \gamma (1 - \alpha) = 0$$

Here the last term is negative since $\alpha > 1$, thus one of the first two terms has to be positive to get 0 in the sum. Notice that these terms can only be positive if $e_G > e_B$ and in this case both of them are positive.

(ii) As $\alpha$ increases, the difference $e_G - e_B$ should increase as well, to get equilibrium.

Regarding the effects of the homophily parameter $\gamma$, observe that as soon as $e_G > e_B$ the ratio $q_B/(\alpha q_G)$ decreases in $\gamma$ (denominator increases, nominator decreases). So the mismatch decreases in $\gamma$.

(iii) The weight on high wages in the conditional expectation of the wages obtained through the family is increasing in $\gamma$ as long as $e_G > e_B$. The weight is given by:

$$\frac{\gamma e_G + (1 - \gamma)e_B}{e_G + e_B} = \frac{\gamma + (1 - \gamma)x}{1 + x}$$

where $x = e_B/e_G$. Derivating with respect to $\gamma$ (taking into account that $x$ changes as well):

$$\frac{1 - x^2 + (1 - 2\gamma)\frac{\partial x}{\partial \gamma}}{(1 + x)^2} > 0$$

This expression is positive as long as $\gamma > 0.5$ since the derivative of $x$ with respect to $\gamma$ is negative (see the previous point). $1 - x^2$ is positive as well. This implies that the family channel gives better offers on averages as the homophily increases.
Now, we show that the family channel provides better offers than the market. Since $\theta = 1$, the weight on good wages in the market expectation is just 0.5. Looking at the family expectation we can see that it is higher for any $\gamma > 0.5$

$$0.5 < \frac{\gamma e_G + (1 - \gamma)e_B}{e_G + e_B} \iff 0.5e_G + 0.5e_B < \gamma e_G + (1 - \gamma)e_B$$

where the inequality hold because $e_G > e_B$.  

### 7.2.6 Corollary 1

Corollary 1 follows from Lemma 2 and Proposition 3.

### 7.2.7 Proposition 4

**Proof** (i) First, we prove that if $\theta > 1$, $e_G > e_B$.

If $k_p = 0$ and $\theta > 1$, the arrival rates of bad and good offers:

$$q_B(u, v, e_B, e_G) = \frac{A u^n}{1 + \theta u}(1 + \gamma e_B + (1 - \gamma)e_G)$$

$$q_G(u, v, e_B, e_G) = \frac{\theta A u^n}{1 + \theta u}(1 + \gamma e_G + (1 - \gamma)e_B)$$

The ratio of bad and good employed in the equilibrium is given by

$$\frac{q_B}{\alpha q_G} = \frac{e_B}{e_G} = \frac{1 + \gamma e_B + (1 - \gamma)e_G}{\alpha(\theta + \gamma e_G + (1 - \gamma)e_B)} \quad (36)$$

After rearranging:

$$0 = e_G - e_B\alpha\theta + (1 - \gamma)(e_G^2 - \alpha e_B^2) + \gamma e_B e_G (1 - \alpha) \quad (37)$$

First, if $\alpha = 1$, the last term on the right-hand side is zero. Assume the contrary, $e_B \geq e_G$. Then both of the remaining terms are negative, the equation cannot hold.

Second, if $\alpha > 1$, the last term is strictly negative. Hence the equality to hold we need that at least one of the first two terms is positive, which is consistent only with $e_G > e_B$ (if $e_B \geq e_G$, they are again negative).

Now, we show that the mismatch $e_B/e_G$ decreases with $\theta$. If $\theta$ increases, $q_G$ increases while $q_B$ decreases. This implies that $e_B/e_G$ has to decrease by the equation (36) which further decreases $q_B$ and increases $q_G$. This latter is because $q_B$ is increasing in $e_B$ and decreasing in $e_G$ as long as $\gamma \geq 0.5$. $q_G$ changes in the other direction.

We use the last equation and express $\theta$ using it:
\[ \theta = \frac{e_G}{\alpha e_B} + \frac{\gamma e_G (1 - \alpha)}{\alpha} + \frac{(1 - \gamma)(e_G^2 - \alpha e_B^2)}{\alpha e_B} \]

If \( \theta \) increases, the right-hand side has to increase as well which is increasing.

This implies that the right-hand side of this expression is negative, then the left-hand side should be negative as well: \( e_G \geq \theta e_B \). This can only be if \( \theta \leq 1 \), the contrary of our assumption that \( \theta > 1 \). Hence, \( e_G > e_B \).

For any equilibrium where \( e_G > e_B \), an increase in \( \gamma \) decreases \( q_B/q_G \) (nominator decreases, denominator increases). Hence with homophily the mismatch decreases.

7.2.8 Proposition 5

Proof First, assume that \( \alpha = 1, \theta > 1, k_p = 0 \).

To compare the expected wages of family and market, we compare the weight on the high wage in the conditional expectation. For the market this is \( \theta/(1 + \theta) \) and for the family:

\[ \frac{\gamma e_G + (1 - \gamma)e_B}{e_B + e_G} = \frac{\gamma + (1 - \gamma)\frac{e_B}{e_G}}{1 + \frac{e_B}{e_G}} \]

First of all, we may see that as \( \gamma \) increases, the market weight does not change, while the family weight increases as \( e_G > e_B \), which is the case if \( \theta > 1 \) (see the previous proposition). As \( \gamma \) changes, \( x \equiv e_B/e_G < 1 \) gets smaller, the derivative of the family weight wrt \( \gamma \):

\[ \frac{1 - x^2 + (1 - 2\gamma)\frac{dx}{dy}}{(1 + x)^2} \]

which is clearly positive as long as \( \gamma > 0.5 \). Hence with homophily the family expected wage improves (this is the same argument as in the proof of Proposition 3).

Second, we show that for \( \gamma = 1 \) the market expectation is the same as the family expectation. We use the equilibrium condition (36) from the previous proof to express \( \theta \) (substituting \( \alpha = 1 \)):

\[ \theta = \frac{(1 - \gamma)(e_G^2 - e_B^2)}{e_B} + \frac{e_G}{e_B} \]

which is equal to \( \frac{e_G}{e_B} \) if \( \gamma = 1 \). This is equal to the family weight on the good wage at \( \gamma = 1 \). Since decreasing \( \gamma \) makes the family expectation worse while does not change the market one, we have the result.

Second, we turn to the case when \( \alpha > 1 \). The weight on the good wage in the family expectation is still increasing in \( \gamma \). But now, we see that for \( \gamma = 1 \), the family expectation is higher...
than that of the market. The weight on good wages in the market expectation is $\theta/(1 + \theta)$, we express $\theta$ using (36).

$$
\theta(\gamma) = \frac{e_G + \gamma e_G e_B(1 - \alpha) + (1 - \gamma)(e_G^2 - \alpha e_B^2)}{\alpha e_B}
$$

Evaluated at $\gamma = 1$ this is:

$$
\theta(1) = \frac{e_G + e_G e_B(1 - \alpha)}{\alpha e_B}
$$

The weight on the good wage in the family expectation is $e_G/(e_G + e_B)$ when $\gamma = 1$. Comparing the two:

$$
\frac{\theta(1)}{\theta(1) + 1} \leq \frac{e_G}{e_G + e_B} \iff \frac{1}{\theta(1)} \geq \frac{e_B}{e_G}
$$

Substituting the value of $\theta$, we obtain:

$$
\frac{1}{\theta(1)} = \frac{\alpha e_B}{e_G(1 + e_B(1 - \alpha))} \geq \frac{e_B}{e_G} \iff \frac{\alpha}{1 + e_B(1 - \alpha)} \geq 1
$$

where the last inequality is indeed true since $\alpha > 1$ which implies that the fraction on the left-hand side has a nominator higher than 1 and a denominator lower than 1. This proves that the family expectation is higher than the market one by $\gamma = 1$. If we decrease the level of $\gamma$, the family expectation decreases.

Now, we evaluate the expectations by $\gamma = 0.5$. The weight on the high wage in the family expectation is 0.5:

$$
\frac{0.5 e_B + 0.5 e_G}{e_B + e_G} = 0.5
$$

while the same weight in the market expectation $\theta/(1 + \theta)$ which is higher than 0.5 if $\theta > 1$. Hence by $\gamma = 0.5$ the market expectation is higher.

Hence, we have seen that by $\gamma = 0.5$ the market expectation is higher and at $\gamma = 1$ the family network’s expectation. By the monotonicity and continuity of the family expectation in $\gamma$, this implies that there is a positive threshold $\tilde{\gamma}_1$ where the two expectations are equal. Above this threshold the family wage is higher.

What remains to show is that the threshold $\gamma$ is decreasing in $\alpha$. The threshold is defined by the equation saying that the two equations are equal:

$$
\frac{\theta}{1 + \theta} = \frac{\tilde{\gamma}_1 (1 - x) + x}{1 + x}
$$

Hence,

$$
\tilde{\gamma}_1 = \frac{\theta}{\frac{1 + x}{1 + \theta} - x}
$$

which is increasing in $x$. If $\alpha$ increase, $x$ decreases, so $\tilde{\gamma}_1$ decreases.
7.2.9 Proposition 6

Proof We use the expressions for the arrival rates as a function of $u$ and $v$ as it is derived in the proof of Proposition 1. We repeat the expressions here for the case of $\alpha = 1$:

\[
q_B(u, v) = \frac{\lambda A v^\beta [1 + (1 - \gamma)(1 - u)]}{\lambda (1 + \theta u) - A v^\beta u (2\gamma - 1)}
\]

(38)

\[
q_G(u, v) = \frac{\lambda A v^\beta [\theta + (1 - u)(1 - \gamma)]}{\lambda (1 + \theta u) - u A v^\beta (2\gamma + P(u) - 1)}
\]

(39)

where $P(u) = (1 - u)\frac{(1 - u)\eta}{u}$ and $P = 0$ if $k_P = 0$, $P > 0$ when $k_P > 0$.

Since $\theta > 1$ already implies the results of the propositions (see the section of the model without professional contacts), it is sufficient if we focus on the case when $\theta = 1$.

If $\theta = 1$, the nominator is the same in the two equations of the arrival rates and the denominator differs only in the term $P$ which makes $q_G > q_B$. Again dividing (20) and (21), we get that

\[
\frac{e_G}{e_B} = \frac{q_G(u, v)}{q_B(u, v)} = \frac{\lambda(1 + \theta u) - A(-1 + 2\gamma)uv^\beta}{\lambda(1 + \theta u) + A(1 - 2\gamma - P)uv^\beta}
\]

The number of the professional contacts enters only through $P$ which is increasing in $k_P$ unless $u = 1$ (which never is the case in equilibrium):

\[
\frac{\partial P(u)}{\partial k_P} = -\frac{(1 - u)^{1+k} \log(1 - u)}{u}
\]

which is positive.

Turning to the derivative wrt the homophily parameter $\gamma$, notice that the derivative both of the nominator and the denominator wrt $\gamma$ is equal to $-2Auv^\beta$. So, we can write the derivative of the fraction with respect to $\gamma$:

\[
\frac{\partial \frac{e_G}{e_B}}{\partial \gamma} = \frac{-2Auv^\beta(\lambda(1 + \theta u) + A(1 - 2\gamma - P)uv^\beta - (\lambda(1 + \theta u) - A(-1 + 2\gamma)uv^\beta))}{(\lambda(1 + \theta u) + A(1 - 2\gamma - P)uv^\beta)^2} = \frac{-2Auv^\beta(Auv^\beta(-P))}{(\lambda(1 + \theta u) + A(1 - 2\gamma - P)uv^\beta)^2}
\]

This expression is clearly positive, hence the ratio $e_G/e_B$ is increasing in $\gamma$.

7.2.10 Corollary 2

Proof This corollary follows from Proposition 6 and Lemma 2.
7.2.11 Proposition 7

Proof (i) Again, we compare the weight on the good wages between the wage expectation of markets and family contacts. For the market this is $\frac{\theta}{1 + \theta}$, which does not change with $\gamma$. For the family we have:

$$\frac{\gamma e_G + (1 - \gamma)e_B}{e_B + e_G}$$

This is decreasing in $\gamma$ if $e_G > e_B$ as has been shown before.

To be able to compare the two quantities we divide the equilibrium conditions (20) and (21) when $\alpha = 1$:

$$\frac{q_G}{q_B} = \frac{e_G}{e_B} = \frac{\theta + \gamma e_G + (1 - \gamma)e_B + e_G P}{1 + \gamma e_B + (1 - \gamma)e_G}$$

where $P \equiv (1 - u)\frac{1 - (1 - u) \rho}{u}$

From here we express $\theta$:

$$\theta = \frac{(1 - \gamma)(e_G^2 - e_B^2) + e_G}{e_B} - e_G P$$

And from here we can express the market weight on good wages:

$$\frac{\theta}{1 + \theta} = \frac{(e_G^2 - e_B^2)(1 - \gamma) + e_G(1 - e_B P)}{(e_G + e_B)(1 + (e_G - e_B)(1 - \gamma)) - e_G e_B P}$$

First we compare the two weights when $\gamma = 0.5$. In this case the market always performs better as long as $\theta > 1$, this was shown in the proof of Proposition 5: the family weight on good wages is 0.5, while the market weight is higher than that.

On the other hand, if $\gamma = 1$, the family expected wage is always higher. Evaluating the two weights at $\gamma = 1$, for the market:

$$\frac{\theta}{1 + \theta} = \frac{e_G(1 - e_B P)}{e_B + e_G(1 - e_B P)}$$

for the family:

$$\frac{e_B}{e_B + e_G}$$

We define the function $f(\gamma) = \frac{\gamma}{\gamma + e_B}$, which is clearly strictly increasing in $\gamma$. Hence we have that $f(e_G) > f(e_G(1 - e_B P))$ since $1 - e_B P < 1$. That is the family weight is higher.

Since the market weight does not change in $\gamma$ while the family weight is increasing, by continuity there is a threshold $\tilde{\gamma}_1 \in (0.5, 1]$ where the two weights are equal and above which the family performs better.

60
By equalizing the two weights on the good wages we find the threshold $\bar{\gamma}_1$:
\[
\bar{\gamma}_1 = \frac{\theta x + 1}{1 + \theta x} - \frac{x}{1 - x}
\]
where $x = e_B/e_G$ which decreases with $k_P$. Hence what remains to show is that the threshold $\bar{\gamma}_1$ is increasing in $x$. The derivative is:
\[
\frac{-1 + \theta}{(1 + \theta)(-1 + x)^2} < 0
\]
which is negative since $\theta > 1$.

As for the overall network (family + professional contacts), the weight on the good wage in the expectation is:
\[
\gamma e_G + (1 - \gamma)e_B + e_GP
\]
where $x = \frac{e_B}{e_G} < 1$ is decreasing in $\gamma$. Hence the weight is increasing in $\gamma$, the derivative is positive:
\[
\frac{-(-1 + x)(1 + P + x) + (-1 + 2(1 + P))\frac{\partial x}{\partial y}}{(1 + P + x)^2} > 0
\]
which is positive since $x < 1$ and $\gamma > 0.5$.

Now we show that for $\gamma = 1$, the network wage is higher than the market: $f(e_G(1 + P)) > f(e_G(1 - e_BP))$ where $f(y)$ is defined as before. As we decrease $\gamma$, the network expectation gets worse, but at $\gamma = \bar{\gamma}_1$ it is still higher than the family expectation:
\[
\frac{\gamma e_G + (1 - \gamma)e_B + e_GP}{e_B + e_G} < \frac{\gamma e_G + (1 - \gamma)e_B + e_GP}{e_B + e_G(1 + P)}
\]
which is true for any value of $\gamma$ including $\bar{\gamma}_1$.

Hence, by continuity, for a $\gamma = \bar{\gamma}_1 - \epsilon$, the family is worse than the market but the overall network is still better.

(iv-v) $\bar{\gamma}_2$ is the solution of the equation:
\[
\frac{\theta}{\theta + 1} = \frac{\bar{\gamma}_2 e_G + (1 - \bar{\gamma}_2)e_B + e_GP}{e_B + e_G(1 + P)}
\]
If $\theta \geq 2$, $\bar{\gamma}_2$ is surely higher than zero. At $\gamma = 0$, the network weight is: $\frac{xP}{1 + xP}$ which is at most 2/3 (since both $P$ and $x$ take their maximum at 1). So the market weight is higher if:
\[
\frac{\theta}{1 + \theta} \geq \frac{2}{3} \iff \theta \geq 2
\]
Expressing the threshold $\bar{\gamma}_2$ from here, we get:
\[
\bar{\gamma}_2 = \frac{\theta}{1 + \theta} \frac{1 + x + P}{1 - x} - \frac{x + P}{1 - x} = \frac{\theta}{1 + \theta} \frac{1}{1 - x} - \frac{1}{1 + \theta} \frac{x + P}{1 - x}
\]
where $x$ is defined as before. If the number of professional contacts increases, $x$ decreases and $P$ increases. Both implies that the threshold decreases: $P$ has negative sign in this expression and if $\theta \geq 2$ the threshold is increasing in $x$. The derivate of $\bar{\gamma}_2$ wrt $x$:

$$\frac{-1 - P + \theta}{(1 + \theta)(-1 + x)^2} > 0$$

where $P$ is at most 1. 

7.2.12 Corollary 3

Proof This corollary follows from Proposition 7 and Lemma 2.

7.3 Figures and Tables

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Table 2: Unemployment rate for different values of $\gamma$ and $k_P$. 
Figure 5: Unemployment rate (1st panel), vacancy rate (2nd panel), fraction of employed in bad jobs (3rd panel), fraction of employed in good jobs (4th panel), wage in bad jobs (5th panel) and wage in good jobs (6th panel) as the function of the number of professional contacts for different values of the homophily index.
Figure 6: Threshold levels of the homophily parameter for different values of $\theta$ and $k_P$ when separation rates are different $\alpha = 2$. Left panel: $\theta = 15$, right panel: $\theta = 30$. 