Tax Morale and Tax Evasion:
Social Preferences and Bounded Rationality

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Abstract. We study a family of models of tax evasion, where a flat-rate tax only finances the provision of public goods. In the first two models, every worker has the same earning, who reports an income so as to maximize his utility, which is a sum of the utility arising from his consumption from after-tax income and of the (moral) utility arising from reported income. The second utility is the product of three factors: the exogenous tax morale, the average reported income observed in his neighborhood and his sub-utility derived from his own report. It can be shown that the stability of the equilibrium of reports is independent of the size of the neighborhood (models A and B). We try to reformulate our principle in the framework of agent-based models (model C). We obtain path-dependence, strange Laffer-curves and the reduction of tax evasion increases with social cohesion.
1 Introduction

In the study of tax systems, tax avoidance and tax evasion should be considered. The first mathematical analyses, Allingham and Sandmo (1972) and Yitzhaki (1974), modeled tax evasion as a gamble: for given audit probability and a penalty proportional to the evaded tax, what share of their income do risk averse individuals report?

Subsequent studies have discovered that the actual probability of audits and the penalty rates are insufficient to explain why citizens of healthier societies pay income taxes in the propensity they do. Of course, the lower the tax rate, the higher the share of reported income. Therefore, another explanatory variable should be introduced, tax morale, as in Frey and Weck-Hannemann (1984). For example, we may assume that every worker has two parameters: his wage and his tax morale. The tax system can also be characterized by two parameters: the marginal tax rate and the cash-back. The tax system has two functions: income redistribution and financing of public goods. We repeat our basic assumption: the reported income is an increasing function of the tax morale and a decreasing function of the marginal tax rate.

Using the classical approach, Simonovits (2010) took tax morality as given, assumed a utility function which took into account the utility derived from reporting income along the utility derived from his own consumption. Specifically, utility increases with tax morale. To avoid perverse cases, Simonovits assumed such a (logarithmic and linear) utility function, which excludes overreporting of income. In his model, taxes finance income redistribution as well as provision of public goods which also generates utility. Obviously, with sufficiently large population, optimizing individuals will not take the latter term into consideration. The optimal report achieves a balance between the larger consumption due to lower report and the moral utility arising from higher report. The major result, so far substantiated only by numerical examples, is that increasing tax morale induces greater income redistribution and net tax, financing more public expenditures.

Traxler (2010) used a more refined model, making the moral utility dependent on the share of norm-followers: the less people cheat, the higher is the moral utility of reporting income. He also incorporated the auditing and penalizing block from Allingham and Sandmo (1972). Now the existence of an equilibrium is quite involved and even the existence of multiple equilibria cannot be ruled out.

At this point, we shortly discuss the approach called agent-based-modeling, for short, ABM. Using examples, Macal and North (2006) summarize the most important properties of this approach. First, one must identify the agents, their classes, equivalent to the description of their states. Then, the neighborhood in which the agents work and interact with each other must be determined. Furthermore, the methods which update the characteristics of the agents and their neighborhoods should be outlined. Finally comes the exact specification of their interactions.
In recent years, economists have started using agent-based models; the economic applications belong to the area of agent-based computational economics. Tesfatsion (2001), Tesfatsion (2006), LeBaron (2006), and Heath et al. (2009) provide good surveys. Their common philosophy is the assumption of bounded rationality, where agents may not optimize an objective function, and understand their neighborhoods imperfectly. Their decisions and interactions are simple and algorithmic, so they can be calculated by computers.

Making a detour, we mention related models in game theory. For example, Ellison (1993) proved that in local interaction games, if players play the risk-dominant strategy with a positive probability, then a large enough population almost surely coordinates on the foregoing strategy. Lee and Valentinyi (2000) replaced mutation by another assumption, namely that at the initial state, every player played the risk-dominant strategy with a positive probability, and proved convergence (cf. Szolnoki).

There are several examples for the agent-based approach to taxation problems, see Bloomquist (2006). These models like others, however, put the main emphasize on the problem of punishing tax evaders. A good outline about the economic applications of ABM is given by Szabó, Gulyás and Tóth (2009), who apply it in modeling Hungarian tax evasion. They build up a complex model, where the employees, the employers and the government interact with each other. For example, if the employee evades tax paying, then the government provides less public services, but if the efficiency of these services is low, then the employee cheats even more. We also mention Bakos, Benczúr and Benedek (2008), which estimated the tax elasticity of taxable income on Hungarian data, using the tax changes in 2005.

The agent-based approach is justified by the possibility of modeling such mechanisms, which resists to analytical approach. On the other hand, a basic insight of the literature is that tax behavior is heterogeneous and cannot be described by simple “selfish” utility maximization. Several papers are characterized by multi-type taxpayers. Hokamp and Pickhardt (2010) is a good example, where there are four types: a rational (traditional selfish utility maximizer), another is a moralist (who pays all his dues on principle), an erratic type (who always makes errors, which is quite relevant, since a non-negligible share of taxpayers overreports its income) and an emulation type (which follows the behavior of others within a social network). (Similar papers: Antunes et al. (2006), Frey and Torgler (2007), Prinz (2010)).

Returning to our models, for the sake of simplicity, we neglect wage differences, as well as income redistribution. Our tax system relies on flat rate without cash-back, therefore we shall drop the adjective marginal from now on.

Every worker has his own time-invariant neighborhood and he is able to observe his neighbors’ tax reports. In the simplest case, he has only two options: either he reports his full income or none of it (Model C). In more sophisticated scenarios, he maximizes his utility function (Model A and B). In both cases we can investigate the existence of an equilibrium or multiple equilibria, and their stability.
Analyzing models A and B, we speak of a symmetrical case if not only the wages but the exogenous tax morales are also identical. For the time being, our most interesting analytical result is that there exists a unique symmetric equilibrium, which is stable in the case of full population observation, and remains stable for partial observation as well.

At a higher level, the government decides on the value of tax rate. Given the individuals’ utility functions, the government can maximize its social welfare function. We encounter with the Laffer-curve, namely the tax revenue first increases then decreases as the tax rate rises.

In our agent-based model (C), income reporting is dichotomous: agents decide on their reports according to a simple rule: if their level of utility is lower than their agent-specific threshold, and if enough of their visible neighbors are tax evaders, they will also evade tax. Otherwise, they report their full income. Similarly to the first models, there is no punishment included in these simulations. The question is whether rational, evasive behavior would become ubiquitous, or everyone will report his full income, or we end up with a polymorphous population. Our results indicate that some, but not all of the tax-evasive behavior is driven out.

If we abstract from audits and penalties, tax compliance is like a public-good provision game. Public Good provision games belong, in their turn, to the family of Prisoner’s Dilemma games, with many players. Here the only strictly dominant strategy is zero contribution. However, in experimental situations the usual finding is substantial positive contribution. This is a robust outcome, in experimental ultimatum, dictatorship or trust games selfish individual rationality is consistently refuted, and some sort of “social behavior” prevails. Though in iterated experiments of Public Good games the level of apparent “benevolence” is diminishing, it is not obvious whether this is due to learning or dissatisfaction of others’ “asocial” behavior, see Fehr and Camerer (2004). These findings corroborate the important role of tax morale, adding some role for the environment, as moral behavior may not be independent of the observation of our neighbors’ acts (Heinrich et al. (2004a), Janssen and Ahn (2003), Cason, Saijo and Yamato (2002)).

We find that the final share of tax evaders is an increasing function of the initial share; and, under most parameter combinations, the share is decreasing in time. The rate of decrease strongly depends on the neighborhood’s size; the larger the neighborhood, the faster the decrease. Social efficiency is also found to be a determinant of tax evasive behavior.

The structure of the paper is as follows: Section 2 outlines the models, while Section 3 presents the results of the simulation. Section 4 concludes. An Appendix models the impact of distributed lags on the convergence speed in a higher-order linear difference equation.
2 The Models

We can apply at least three approaches. a) The traditional neoclassical one, where individuals maximize their morale-dependent utility function, decide on their optimal reports; but the utility functions also depend on the average reports (proportional to the public expenditures) in the previous period. b) The bounded rationality assumption, where the individuals only observe their narrow neighborhood, but still optimize their utility functions. c) Not only the neighborhood is narrow but the decision is also boundedly rational. We shall present the three approaches one after another.

2.1 Model A: Full observation, utility maximization

Let the number of workers be a positive integer $I$ and index the workers as $i = 1, 2, \ldots, I$. Let the index of periods (say years) be $t = 0, 1, 2, \ldots$. We assume that every worker has the same earnings, for simplicity, unity: $w = 1$. Then there is no reason for income redistribution, the tax only finances the provision of public goods.

Let $m_i$ be worker $i$’s exogenous tax morale, let $v_{i,t}$ be his income report in period $t$, $v_m \leq v_i \leq 1$, where $v_m$ is the minimal report, $0 \leq v_m < 1$. (By the logic of the theory, the government also knows that everybody earns a unity, nevertheless, it tolerates underreporting, it only insists on a minimal report.)

In period $t$ the average reported wage is equal to

$$\bar{v}_t = \frac{\sum_{i=1}^{I} v_{i,t}}{I}.$$ 

Let $\theta$ be a real number between 0 and 1, the tax rate. Let $c_{i,t}$ denote the $i$’s worker consumption: $c_{i,t} = 1 - \theta v_{i,t}$, its traditional utility function is then $u(c_{i,t})$. Let the moral utility function be $z(v_{i,t}, m_i, \bar{v}_{t-1})$, which has three components: utility derived from his own report $v_{i,t}$, his exogenous morale $m_i$ and $\bar{v}_{t-1}$. (The exogenous tax morality has no direct economic meaning, it can only be inferred from indirect observations.) Finally let the per capita public expenditure be $X_t$, i.e. $X_t = \theta \bar{v}_t$, whose individual utility is $q(X_t)$.

Let us assume that $u(\cdot)$, $z(\cdot, \cdot, \cdot)$ and $q(\cdot)$ are strictly increasing and strictly concave function. The worker $i$’s utility is the sum of three terms:

$$U_{i,t} = u(c_{i,t}) + z(v_{i,t}, m_i, \bar{v}_{t-1}) + q(X_t).$$

Of course, maximizing $U_i$, the worker neglects the third term, because this depends on the simultaneous decisions of many other workers. In other words, in period $t$ worker $i$ reports such an income which maximizes his narrow utility:

$$U_{i,t}^*(v_{i,t}) = u(1 - \theta v_{i,t}) + z(v_{i,t}, m_i, \bar{v}_{t-1}) \to \max.$$ 

In case of interior optimum,

$$U_{i,t}^{*\prime}(v_{i,t}) = -\theta u'(1 - \theta v_{i,t}) + z'(v_{i,t}, m_i, \bar{v}_{t-1}) = 0,$$
where \( z_1' \) is the partial derivative of function \( z \) with respect to the first variable.

In case of a corner solution, either

\[
v_{i,t} = v_m \quad \text{if} \quad U_{i,t}^{*'}(v_m) = -\theta u'(1 - \theta v_m) + z_1'(v_m, m_i, \bar{v}_{t-1}) < 0,
\]

or

\[
v_{i,t} = 1 \quad \text{if} \quad U_{i,t}^{*'}(1) = -\theta u'(1 - \theta) + z_1'(1, m_i, \bar{v}_{t-1}) > 0.
\]

The system’s initial state is \( v_0 = (v_{i,0}, \ldots, v_{I,0}) \). The system is in an equilibrium, if starting the system from it, the system remains there. Notation: \( v^o = (v^o_1, \ldots, v^o_I) \)

The system is called asymptotically stable if (i) for any real \( \varepsilon > 0 \) there exists another real \( \delta > 0 \), such that if the initial state is in the \( \delta \)-neighborhood of the equilibrium, it stays in the \( \varepsilon \)-neighborhood of the equilibrium and (ii) \( v_{i,t} \) asymptotically converges to \( v^o_i \).

The social welfare function is given by

\[
V = \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{I} U_{i,t},
\]

where real \( \beta \) is between 0 and 1, the discount factor.

We need apply parameterized rather than general utility functions, c.f. Simonovits (2010).

\[
u_i(c_{i,t}) = \log c_{i,t}, \quad z(v_{i,t}, m_i, \bar{v}_{t-1}) = \bar{v}_{t-1}m_i(\log v_{i,t} - v_{i,t}), \quad q(X_t) = \omega \log X_t,
\]

where real \( \omega > 0 \) is the efficiency parameter of the public expenditures. Note that the first and second factors of \( z \) are endogenous and exogenous tax morale, while the third factor in brackets increases with reported income \( v_{i,t} \) for \( v_{i,t} < 1 \) and decreases otherwise: there is no moral urge to overreport income.

Then the implicit difference equation of the optimal report is

\[-\frac{\theta}{1 - \theta v_{i,t}} + \bar{v}_{t-1}m_i\left(\frac{1}{v_{i,t}} - 1\right) = 0, \quad i = 1, 2, \ldots, I.\]

Making it explicit, and solved,

\[
v_i = \frac{\theta + m_i(\theta + 1)v_{t-1} - \sqrt{[\theta + m_i(\theta + 1)v_{t-1}]^2 - 4m_i^2\theta v_{t-1}^2}}{2m_i\theta v_{t-1}}.
\]

Finally, it is worth displaying the equation of the equilibrium, at least for the symmetric case \( m_i = m \):

\[-\frac{\theta}{1 - \theta v^o} + v^o m\left(\frac{1}{v^o} - 1\right) = 0.
\]
Then $v^o = 1 - \theta v^o$, $z^o = v^o m \log v^o$ and $q(X^o) = \omega \log(\theta v^o)$.

It can be shown that the symmetric equilibrium of reports is the lower root of a quadratic equation:

$$v^o = \frac{(1 + \theta) - \sqrt{(1 - \theta)^2 + 4\theta^2/m}}{2\theta}.$$  

It is easy to see that the equilibrium report is an increasing function of the exogenous morale. It is more difficult to prove that the equilibrium report is a decreasing function of the tax rate.

When does an equilibrium exist? If $v_m \leq v^o \leq 1$, i.e.

$$-\frac{\theta}{1 - \theta v_m m} + v_m m \left(\frac{1}{v_m} - 1\right) > 0, \text{ i.e. } m > \frac{\theta}{(1 - \theta v_m)(1 - v_m)}$$

In principle, we can determine the socially optimal tax rate which maximizes $V$, or $U_i$:

$$U_i(\theta) = \log(1 - \theta v^o(\theta)) + v^o(\theta) m(\log v^o(\theta) - v^o(\theta)) + \omega \log(\theta v^o(\theta)).$$

At this point we introduce the famous Laffer-curve, which depicts the tax revenue as a function of the tax rate:

$$X(\theta) = \theta v^o(\theta).$$

It can be shown that the Laffer-curve is increasing for low tax rates and decreases for high rates assuming $m < 6 + 2\sqrt{5} \approx 10.5$.

Returning to the dynamics, note that after one step, the non-equilibrium path also becomes symmetric: $v_{i,t} = v_t$, because everybody will have the same optimization problem. Dropping index $i$,

$$F(v_{t-1}, v_t) = -\frac{\theta}{1 - \theta v_t} + v_{t-1} m \left(\frac{1}{v_t} - 1\right) = 0.$$  

The sufficient condition of local asymptotic stability is $|F'(v^o)| < 1$, where by the theorem on implicit function,

$$\lambda = \frac{dv_t}{dv_{t-1}} = -\frac{F'_1}{F'_2}. $$

Here $F'_1$ and $F'_2$ are the partial derivatives with respect to variable 1 and 2, respectively.

$$F'_1 = m \left(\frac{1}{v^o} - 1\right) \text{ and } F'_2 = \frac{-\theta}{(1 - \theta v^o)^2} + m \frac{1}{v^o}.$$  

With arrangement, the local stability condition is

$$0 < \theta < m(1 - \theta v^o)^2$$

The issue of global stability requires further analysis.
2.2 Model B: Limited Observation, Utility Maximization

We replace now the assumption of model A about the global observability by the local observability. Let $N_i$ be worker $i$’s neighborhood (the index set of its neighbors), which is a small subset of the total index set $N = \{1, 2, \ldots, I\}$. Let $\bar{v}_{i,t}$ be the average report of $i$’s neighborhood in time $t$:

$$
\bar{v}_{i,t} = \frac{\sum_{j \in N_i} v_{i,t}}{|N_i|},
$$

where $|N_i|$ is the number of neighbors, a small number relative to $I$, and in a symmetric case independent of $i$. For example, $|N_i| = d \ll I$.

Because of this generalization, we display several formulas again, in the general frame:

$$
U_i^{*'}(v_{i,t}) = -\theta u'(1 - \theta^2 v_{i,t}) + z_i'(v_{i,t}, m_i, \bar{v}_{i,t-1}) = 0,
$$

where $z_i'$ is the partial derivative of $z$ with respect to the first variable.

In case of corner solution, either

$$
v_{i,t} = v_m \text{ if } U_i^{*'}(v_m) = -\theta u'(1 - \theta v_m) + z_i'(v_m, m_i, \bar{v}_{i,t-1}) < 0,
$$

or

$$
v_{i,t} = 1 \text{ if } U_i^{*'}(1) = -\theta u'(1 - \theta) + z_i'(1, m_i, \bar{v}_{i,t-1}) > 0.
$$

Then the equation for the optimal report (for interior optima) is

$$
-\frac{\theta}{1 - \theta \bar{v}_{i,t}} + \bar{v}_{i,t-1} m_i \left( \frac{1}{v_{i,t}} - 1 \right) = 0.
$$

We often assume that the model is symmetric, i.e. if arbitrarily permute the individuals’ indices, then the neighborhoods are also permuted correspondingly. Specifically, for every $i$, $|N_i| = d$ and $m_i = m$.

For the time being, we consider the simplest case.

**Example 1.** The workers are located on a circle: with convention $I + 1 = 1$ and $0 = I$, $N_i = \{i - 1, i + 1\}$. Then $\bar{v}_{i,t-1} = (v_{i-1,t-1} + v_{i+1,t-1})/2$, i.e. at an equilibrium $v_0^*$ as in model A.

The symmetric equilibrium is unchanged and the existence of an asymmetric one seems to be implausible.

We outline the proof of our conjecture now. We assume the contrary, i.e. there exists an equilibrium vector, such that at least two subsequent elements are different: $v_0^* \neq v_2^*$, e.g. $v_1^* < v_2^*$. For $I = 3$, we may assume that $v_0^* < v_2^*$. Due to $v_0^* = F((v_0^* + v_2^*)/2)$ and $v_2^* = F((v_0^* + v_2^*)/2)$, and monotonicity, we have $v_0^* + v_2^* < v_1^* + v_3^*$, i.e. $v_0^* < v_1^*$, a contradiction.

For $I = 4$, $v_2^* = F((v_0^* + v_2^*)/2) = v_1^*$, i.e. $v_2^* = v_1^*$. Due to symmetry, $v_1^* = v_3^*$, inserting back: $v_2^* = F(v_1^*)$ and $v_1^* = F(v_2^*)$, contradicting to the monotonicity of $F$. 


To study local stability, we shall need the general entry of the matrix of the linearized equation \( \mathbf{v}_t = \mathbf{A} \mathbf{v}_{t-1} \):

\[
a_{ij} = \begin{cases} 
\frac{F'(v^o)}{2}, & \text{if } j = i - 1 \text{ or } j = i + 1; \\
0, & \text{otherwise}.
\end{cases}
\]

Let \( \mu = \frac{F'(v^o)}{2} \). For \( I = 3 \), matrix

\[
\mathbf{A} = \begin{pmatrix}
0 & \mu & \mu \\
\mu & 0 & \mu \\
\mu & \mu & 0
\end{pmatrix}.
\]

(In this case, the exclusion of game theoretic interaction is unjustified, but we only display it as an illustrative example.) Local stability can again be investigated analytically. The characteristic polynomial of matrix \( \mathbf{A} \) is

\[
P(\lambda) = \lambda^3 - 3\mu^2 \lambda - 2\mu^3,
\]

which has three real roots: \( \lambda_{1,2} = -\mu \) (with multiplicity 2, since \( P'(-\mu) = -3\mu^2 + 3\mu^3 = 0 \)) and \( \lambda = 2\mu \), the dominant root. System B is stable if and only if \( F'(v^o) = 2\mu < 1 \).

Books on matrices also contain the general solution and outline the connection with random walks. It is easy to show that \( \lambda = 2\mu \) remains a root, but its dominance is difficult to show. In the simulation part, we shall also show that the stability of Model A is inherited by Model B.

For example, with the help of the max vector norm and the sum matrix norm it can be demonstrated that for \( \mu < 1/2 \), the stability of the symmetric equilibrium remains valid for any \( I \). Moreover, if the the number of neighbors \( d \geq 2 \), then \( \mu < 1/d \) is a sufficient condition for stability. (The discussion of necessary and sufficient condition is left for further investigations.)

2.3 Model C: Limited Observation, Decisions Based on Simple Heuristics

Overview

There are \( I \) agents, having uniform real income (measured in a “private” good) in each period \( t \), which is normalized to 1. The individual budget constraints are:

\[
c_{it} + \tau_{it} = 1,
\]

where \( c_{it} \) private consumption, and \( \tau_{it} \) taxes paid. Agents have identical utility functions:

\[
\begin{equation}
\begin{aligned}
\ell_{it} &= \log(1 + c_{it}) + \omega \log(1 + X_t),
\end{aligned}
\end{equation}
\]

where

\[
\sum_{i=1}^{I} \tau_{it} = \frac{X_t}{I}
\]
represents the service of the public good enjoyed by an agent. \( \sum_{i=1}^{I} \tau_{i,t} \) is the amount of the public good financed by tax.) Each agent \( i \) has a neighborhood \( N_i \subset I \), interpreted as the set of "acquaintances" whose behavior is observable by \( i \).

The tax behaviour of each agent is binary variable:

\[ \tau_{i,t} = 0 \]

if agent \( i \) does not pay taxes in period \( t \), and

\[ \tau_{i,t} = \theta, \]

if he does.

We consider compliance as a "social norm": agents behave according to this norm, except when they have strong causes to deviate. Agents in our model deviate, i.e. do not pay taxes, whenever they are sufficiently frustrated by the system. Their frustration is a combination of internal and external factors.

Each agent \( i \) is characterized by a real number \( u_i \) giving his frustration threshold in terms of utility. We say that an agent is "internally" frustrated in \( t \) if

\[ u_{i,t-1} < u_i. \]

Agents with higher \( u_i \) become more easily dissatisfied with society. Notice that a high \( \omega \), which is a proxy for the quality of public goods, diminishes the chances of internal frustration.

However, external frustration is only necessary, but not sufficient for deviance. There is another necessary condition for this, which is external and local, rather than internal: agents also observe their neighbor's behavior. If they find that many of them do not comply with the social norm, they become "externally" frustrated. We speak about a tolerance level \( m_i \) \((0 \leq m_i \leq 1)\) for each agent, and if

\[ m_i < \frac{|N_i \cap \{j \mid s_{j,t-1} = 0\}|}{|N_i|} \]

then we say that agent \( i \) in period \( t \) is externally frustrated. With a low level of tolerance an agent breaches the social norm more easily. The basic assumption of our model is that the combination of these two causes, internal and external frustration, is necessary and sufficient for deviant behavior. Let \( 0 < \theta < 1 \) be the tax rate. Then the behavior of an agent in \( t \) is simply:

\[ \tau_{i,t} = \begin{cases} 0 & \text{if } u_{i,t-1} < u_i \text{ and } m_i < \frac{|N_i \cap \{j \mid s_{j,t-1} = 0\}|}{|N_i|} \\ \theta, & \text{otherwise} \end{cases} \]

\( \tau_{i,t} \) is also the (partly) public state of agent \( i \) in period \( t \), inasmuch as those people whose neighborhoods contain \( i \) can observe it. The agent characteristics \((u_i, m_i)\) are time invariant, and non-observable. People are not "good" or "bad"
by themselves, their behavior, however, can be moral or immoral, depending on their disposition, and on external circumstances. To diminish the significance of the initial pattern of tax compliance, we also introduced "noise" in the system. That is, each turn agents have a fixed chance of randomly switching from tax evasion to compliance or vice-versa.

Tax compliance models usually include audits and penalties, as means to compel otherwise selfish agents to comply. We are well aware that real societies use punishment as an incentive device. Therefore our model is purely theoretical, it asks a simple question: can the social norm of tax compliance prevail in a society that is characterized by the properties described above? To answer this question we run simulations, where we observe the path of aggregate compliance starting from arbitrary initial levels.

3 Simulations

Even in the simplest cases, it is worth making numerical calculations, to shed light on the order of magnitudes.

3.1 Model A

We start our investigations by experimenting with the parameter pairs \((m, \theta)\): Table 1. First, we get rid of those pairs which generate optimal reports below the minimum or above 1. For lower tax morale \((m = 0.5)\), lower optimal tax rates (about \(\theta = 0.25\)) are obtained, for higher tax morales \((m = 1)\), higher optimal tax rates (about \(\theta = 0.5\)) are obtained. It is remarkable that for low as well as high tax morales, the Laffer curve is already decreases around the optimum: for \(m = 0.5\), raising the tax rate from 0.2 to 0.25, the tax revenue drops from 0.11 to 0.1; while for \(m = 1\), raising the tax rate from 0.45 to 0.5, the tax revenue drops from 0.198 to 0.191, while the welfare increases. Notwithstanding, it is still a paradox that the socially optimal report is a decreasing function of the exogenous tax morality.

3.2 Model B

For symmetric initial states the difference between models A and B immediately disappears, but it survives for asymmetric initial states. The stability of models A survives.

We select the pair \(m = 1\) and \(\theta = 0.5\) and analyze the dynamics of asymmetric initial states for three agents. Let \(x = 1.3\), \(v_{1,0} = xv^o\), \(v_{2,0} = v^o\) and \(v_{1,0} = v^o/x\). It is sufficient to display the first four state vectors to see the preservation of stability: Table 2.
Table 1. The impact of morale and tax rate

<table>
<thead>
<tr>
<th>Morale (m)</th>
<th>Tax Rate (θ)</th>
<th>Report (v)</th>
<th>Tax (X)</th>
<th>Utility (U)</th>
<th>Expansion (λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>0.551</td>
<td>0.110</td>
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<td>-0.623</td>
</tr>
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<td>0.110</td>
<td>-2.604</td>
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</tr>
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<tr>
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<td>0.50</td>
<td>0.382</td>
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</table>

Fig. 1. Laffer-curves

Table 2. Limited observation also stabilizes

<table>
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<th>Period (t)</th>
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<th>report 2 (v₂)</th>
<th>report 3 (v₃)</th>
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</table>
3.3 Model C

The Algorithm

1. Define the parameters $(I, \omega, \theta)$.
2. Determine the neighborhoods $N_i \in \{1, \ldots, I\}$ for each $i$.
   This step amounts to setting up a graph, for the details see below.
3. Generate random $u_i$ values. The dissatisfaction threshold has a uniform distribution on $(0, \log(4))$.
4. Generate random tolerance levels $m_i$, uniformly distributed on $(0, 1)$.
5. Determine initial strategies: let $\tau_{i0} = \theta$ with probability $p$.
6. Calculate $u_{i,t}$.
7. Determine $\tau_{i,t+1}$.
8. Let $t = t+1$
9. Iterate on 6, 7 and 8 until the system becomes stationary.

Results

We chose the noise parameter to be 0.05, so that on average, 5 percent of the population switches strategies randomly. For each parameter combination the system converged in finite steps either to a micro stationary state. Therefore in the following we report numerical values for averages of the last ten periods of the relevant macro state variable, the average number of tax evaders. To further diminish the effects of initial states, we averaged out results of 50 runs for each parameter vector.

Tables 3 and 4 illustrate the effect of $d$, social cohesion. Stronger cohesion clearly implies less cheating eventually. On the other hand, one can observe “diminishing returns” to cohesion. For cross effects that higher social efficiency ($\omega$) slightly diminishes the effect of cohesion. Both Tables 4 and 5 indicate that public efficiency reduces cheating. This appears as particularly forceful when there are many cheaters initially, and when social cohesion is strong. Table 5 also indicates some dependence on the initial state, decreasing with public efficiency.

From an economic point of view it is an important question to ask what kind of Laffer-curves emerge from our model. Figs. 2 and 3 show Laffer-curves — the relationship between the tax rate ($\theta$) and tax revenues — for different $\omega$s. From the interpretation of $\omega$ it is expected that Laffer-curves corresponding to higher social efficiency dominate Laffer-curves with lower social efficiency. What is surprising is the shape of the Laffer-curves: they are monotonously increasing on the whole interval. It is also apparent that, with a high value for the social cohesion parameter ($d = 10$), efficiency becomes almost irrelevant: agents then are able to enforce the social norm, regardless of how much of the public good is wasted.

To summarize and check these findings we ran linear regressions with the end-state and the tax income as dependent variables, and the initial state, the tax rate, social cohesion, and social efficiency as regressors. As Tables 6

---

4 $\log(1 + 1) + \log(1 + 1) = \log(4)$ is the maximum attainable utility: the agent does not pay taxes, everybody else does, and the creation of public good is maximally efficient.
and 7 show, our former observations seem to be validated. It is also clear why the Laffer-curves are monotonous on the whole interval: although increasing the tax rate $\sigma$ also increases tax evasion, it does so to a very small extent. Thus, increasing the tax rate by 10 percentage points induces a 9.4 percentage point increase in tax income, ceteris paribus.

### Table 3. Greater social cohesion decreases tax evasion

<table>
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### Table 4. Greater social cohesion and efficiency decrease tax evasion

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### 4 Conclusions

We have analyzed the three models of reporting incomes: A) global observation and utility maximization; B) local observation and utility maximization; C) local observation and decisions based on simple heuristics. Unfortunately, models A and B are so much simplified that we had to limit the parameter space artificially.
Table 5. The effect of the initial state and efficiency on tax evasion

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Fig. 2. Laffer-curves for different values of the efficiency parameter, $d = 1$

Fig. 3. Laffer-curves for different values of the efficiency parameter, $d = 10$
Table 6. Regression coefficients and $R^2$ for explaining the percentage of tax evaders

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<th>t-stat</th>
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Table 7. Regression coefficients and $R^2$ for explaining tax income

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<td>$c$</td>
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<tr>
<td>$R^2$</td>
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</table>

to eliminate overreporting incomes. Small wonder that our analytical results are quantitatively distorted: stable equilibrium only exist for unrealistically high tax rates and unrealistically low consumption. The results obtained in the agent-base model complement those derived above: it turns out that the degree of tax evasion diminishes in time, and its rate is increasing with the size of the neighborhood.

5 Appendix: The impact of distributed lags on the convergence speed

In the main text, the impact of the size of the neighborhood on the convergence speed plays an important role. In this Appendix, we consider a simpler, though related issue: how does the distributed lag influence the convergence speed?

Let $r$ be a positive integer and $y_t$ be the scalar state of the system at period $t$, $t = 0, 1, \ldots$. For distributed lags, the linear system’s dynamic is described by a difference equation of order $r$:

$$y_t = \sum_{i=1}^{r} \alpha_i y_{t-i},$$

where $\alpha_i$, $i = 1, \ldots, r$.

As is known, the system’s asymptotic convergence speed is given by the reciprocal of the dominant characteristic root $\lambda$, where $\nu = 1/\lambda$ satisfies the characteristic equation

$$1 = \sum_{i=1}^{r} \alpha_i \nu^i.$$
If $\alpha_i \geq 0$, then the dominant root is positive and it is the unique positive root. We vary the value of $r$, and fix the total impact, we equalize all the coefficients as $\alpha_i = \alpha/r$, and investigate the dependence of $\nu_r$ on $r$.

**Theorem A.1.** If the total impact is fixed, the distributed coefficients are equal and the system is stable, then the convergence speed $\nu_r$ is a decreasing function of the maximal lag: $1 < \nu_{r+1} < \nu_r$.

**Proof.** By contradiction, assume that there is a positive integer $r$ such that $1 > \nu_{r+1} \geq \nu_r$. Equalizing the right hand sides of the two characteristic equations,

$$\frac{\alpha}{r+1} \sum_{i=1}^{r+1} \nu_{r+1}^i = \frac{\alpha}{r} \sum_{i=1}^{r} \nu_r^i.$$

Introducing the notation $S = \sum_{i=1}^{r} \nu_r^i$, our last equation can be transformed into an inequality

$$r(S + \nu_{r+1}^{r+1}) \leq (r+1)S, \quad \text{i.e.} \quad r\nu_{r+1}^{r+1} \leq S.$$ 

But the sequence $\nu_{r+1}^{r+1} > 1$ is increasing, therefore an opposite inequality holds, yielding a contradiction.

Next we illustrate the order of magnitudes. Let $\alpha = 0.5$ and using the sum of the geometric progression, the characteristic equation reduces to

$$1 = \frac{\alpha\nu \nu_r - 1}{\nu - 1},$$

or

$$F(\nu) = \alpha\nu^{r+1} - (\alpha + r)\nu + r = 0.$$ 

Using the Newton-method, the dominant root is given as a limit of the sequence

$$\nu(k + 1) = \nu(k) - \frac{F(\nu(k))}{F'(\nu(k))}, \quad k = 0, 1, 2, \ldots$$

and initial condition $\nu(0) = 1/\alpha$.

<table>
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<tr>
<th>Lag</th>
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Table 8. Lag and Convergence speed

In fact, by increasing the maximal lag, the convergence speed strongly diminishes.
References


