An Estimated Euro-area DSGE model with Financial Frictions: Empirical Investigation of the Financial Accelerator Mechanism*

Paolo Gelain†
Diego Rodríguez Palenzuela‡
Balázs Világi§

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Abstract

In this paper we combine the seminal euro-area DSGE model of Smets and Wouters (2003, 2005, 2007) with the *financial accelerator mechanism*, that is, the enhanced response of investments to shocks due to the counter-cyclical behaviour of the financial premium, presented in Bernanke et al. (1999), and consider the consequences of taking the augmented model to euro-area data. Our empirical investigation reveals that the particular form of the financial accelerator model we used has several shortcomings.

First, we demonstrate that the structural restrictions derived from the theoretical financial accelerator model are not supported by the data. If all structural restrictions of the model are applied then the pure Smets – Wouters model without any financial frictions fits the data better. Alleviating some of these restrictions significantly alters the value of key estimated parameters and improves the ability of the model to explain the data.

Second, we find that although the theoretical financial accelerator model endogenously determines the external finance premium its empirical version is quit weak to explain empirical fluctuations of the premium.

Finally, we show that the financial accelerator mechanism is not robust to some empirically relevant parameter values.

Keywords: DSGE models, Financial accelerator mechanism, Bayesian Estimation
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†Post-doctoral research fellow, School of Economics and Finance, University of St Andrews, e-mail: paologelain@yahoo.it.

‡Head of Capital Markets & Financial Structures at the European Central Bank, e-mail: diego.rodriguez@ecb.europa.eu.

§Principal researcher at the Magyar Nemzeti Bank (Central Bank of Hungary), e-mail: vilagib@mnb.hu.

1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have become the workhorse of macroeconomics in the past decade, although, the most popular DSGE models neglect financial frictions, see, e.g., Christiano et al. (2005) and Smets and Wouters (2003, 2005, 2007, SW). However, the recent financial crises made it clear that this strategy cannot be continued anymore; DSGE models invented to study financial imperfections, such as Bernanke et al. (1999, BGG) and Iacoviello (2005), have been appreciated.

The objective of this paper is to investigate the empirical performance of the *financial accelerator* (FA) mechanism developed in BGG. In the FA model it is assumed that due to asymmetric information financing a project by external funds requires to pay an extra *financial premium* and the behaviour of the financial premium is explained endogenously by the leverage ratio, that is the ratio of entrepreneurs' net worth to capital stock.

We complemented the model of SW with financial imperfections presented in BGG, and using euro-area macroeconomic and financial data, we estimated the combined model (SWFA) by Bayesian-techniques. We focused on the following questions. First, can adding financial imperfections to the SW model improve its ability to explain the data? Second, are the structural restrictions derived from the theoretical FA model supported by the data, and do these restrictions help the model to fit the data? Third, can the endogenous mechanism of the FA model explain the empirical behaviour of the financial premium? Finally, is the FA mechanism described in BGG robust to different parameter values?

Similar empirical analyses have already been done for a handful of countries.[1] However, our paper has two important distinctive features. Unlike majority of the existing literature, we use financial information, a proxy for the euro-area corporate bond spread, beyond the standard macroeconomic variables to assess the model. Furthermore, we investigate carefully the validity of structural restrictions derived from the FA model and their role in the empirical performance of the model.

Our empirical investigation reveals that the particular form of the financial accelerator model we used has several serious shortcoming if it is taken to the data.

We demonstrate that the structural restrictions derived from the theoretical financial accelerator model are not supported by the data. If all structural restrictions of the BGG model are applied then even the pure SW model without any financial frictions fits the data better. Alleviating some of these restrictions significantly alters the value of key estimated parameters and improves the ability of the model to explain the data. Moreover, one can find such a version of the SWFA model which beats the empirical performance of the pure SW model.

A possible interpretation of the above findings is that the qualitative features of the financial accelerator mechanism has the potential to improve standard DSGE models without financial frictions, however, its structural restrictions are too strong to apply their numerical implications. Probably, the problem with the structural restrictions are related to the restrictive assumptions introduced in BGG for the sake of tractability.

We also find that although the theoretical financial accelerator model endogenously determines the external finance premium, the SWFA model is quit weak to explain empirical fluctuations of the premium. Most of the variance of the premium is explained by an exogenous financial shock.

Finally, we show that the financial accelerator mechanism, the enhanced response of investments to shocks due to the counter-cyclical behaviour of the financial premium, is not robust to some empirically

[1]See, e.g., Gelain and Kulikov (2009) on Estonia, De Graeve (2008) on the US, Christiano et al. (2008) on the US and the euro-area, Lopez and Rodriguez (2008) on Colombia, Christensen and Dib (2008) on Canada, Elekdag et al. (2005) on Korea, Queijo (2005, 2008) on the US and the euro-area, Gelain (2009) on the euro-area, Meier and Müller (2005) on US, Gilchrist et al. (2009) on the US.

relevant parameter values. In the presence of most estimated shocks the FA mechanism does not work in that version of our estimated model which fits the data best.

The paper is structured as follows. *Section 2* presents the model. *Section 3* describes the data and the methodology we used in estimating the model. In *section 4* we presents estimation results. In *section 5* the properties of the estimated model is studied by variance decomposition and impulse response analysis. Finally, *section 6* concludes.

2 The Model

Our model combines the DSGE model of Smets and Wouters (SW, 2003, 2005 and 2007) with the financial accelerator mechanism developed in Bernanke, Gertler and Gilchrist (BGG, 1999).

The basic structure of the model is as follows: there are five types of agents, called households, retailers, intermediate goods producers, capital goods producers and entrepreneurs. While retailers, intermediate goods producer and capital goods producer firms are owned by households, entrepreneurs are distinct entities in order to explicitly motivate lending and borrowing.

Entrepreneurs play a key role in modeling the phenomenon we refer to as financial accelerator. They borrow from a financial intermediary that converts household deposits into business financing for the purchase of capital. Due to the presence of asymmetric information external finance require an extra premium, hence entrepreneurs' demand for capital goods depends on their financial position.

Households are infinitely lived dynasties who consume, save, supply labour and set (sticky) wages in a monopolistically competitive market. Final goods are used for consumption and investments and produced by retailers and intermediate goods producers. Physical capital is provided by entrepreneurs and capital goods producers.

2.1 Households

Optimization problem of the representative household

The domestic economy is populated by a continuum of infinitely-lived households. The expected utility function of household i is

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\varepsilon_t^b \left\{ U(H_t(i), L_t(i)) \right\} \right], \tag{1}$$

for all $i \in [0,1]$. $H_t(j) = C_t(i) - hC_{t-1}$, where $C_t(i)$ denotes the consumption of household i at date t, C_{t-1} is aggregate consumption at date $t-1$, parameter $h \in [0,1]$ measures the strength of external habit formation, $L_t(i)$ is the labor supply of household i. Furthermore, $U(H,L) = H^{1-\sigma_c}/(1-\sigma_c) - L^{1+\sigma_l}/(1+\sigma_l)$ and σ_c, $\sigma_l > 0$, $\mathcal{E}_m > 0$ are parameters of the utility function. ε_t^c is a preference shocks governed by an exogenous AR1 process. As a technical matter, instead of ε_t^b its transformation, ε_t^c, is estimated. It is assumed that $\log \varepsilon_t^c = (1-h) \left[\sigma_c \left(1+h\right) \right]^{-1} \log \varepsilon_t^b$ and $\log \varepsilon_t^c = \rho_b \log \varepsilon_{t-1}^c + u_t^c$ with iid. $u_t^c \sim N\left(0, \sigma_c^2\right)$.

The intertemporal budget constraint of a given household can be written in the form

$$P_t C_t(i) + \frac{B_{t+1}(i)}{R_t^n} = B_t(i) + \mathcal{X}_t^w(i) + W_t(i) L_t(i) + Div_t - T_t, \tag{2}$$

where P_t is the consumer price index, $B_{t+1}(i)$ is the household's holding of riskless nominal bonds at the beginning of time period t, R_t^n is the corresponding one-period gross nominal interest rate, Div_t denotes dividends derived form firms. It is assumed that dividends are equally distributed among households. T_t denotes the lump-sum tax levied on households and $W_t(i)$ is the nominal wage paid to household i. Households supply differentiated labor, hence the wage paid to individual households can be different. On the other hand, $\mathcal{X}_t^w$ is a state-contingent security which eliminates the risk of heterogeneous labor supply and labor income. An optimizing household chooses the trajectory of its consumption, bond-holding, investments, physical capital and capital utilization. It is assumed that a certain household supplying type j of labor belongs to a trade-union representing the interest of optimizing and non-optimizing households supplying type j of labor. The union determines the labor supply and the nominal wage of its members, all members accept its decision.

The formal optimization problem of the households is the following: they maximize the objective function (1) subject to the budget constraint (2) , non-negativity constraints on consumption and investments, and no-Ponzi schemes.

Due to the existence of asset $\mathcal{X}_t^w$ the wage incomes of all households are the same. As a consequence, all households choose the same consumption allocation. We, therefore, drop index i from subsequent notations.

The first order conditions with respect to C_t and B_{t+1} are

$$\varepsilon_t^b \left(C_t - hC_{t-1}\right)^{-\sigma_c} - \lambda_t P_t = 0,$$
$$\beta^t \lambda_t - \beta^{t+1} \mathrm{E}_t \left[\lambda_{t+1} \frac{R_t^n}{\Pi_{t+1}}\right] = 0,$$

where and $\Pi_t = P_t/P_{t-1}$ denotes CPI inflation and $\beta^t \lambda_t$ is the state contingent lagrangian multiplier of the budget constraint (2). Combining the above two conditions yields the consumption Euler equation

$$\Lambda_t = \beta \mathrm{E}_t \left[\frac{R_t^n}{\Pi_{t+1}} \Lambda_{t+1}\right], \tag{3}$$

where $\Lambda_t = \varepsilon_t^b \left(C_t - hC_{t-1}\right)^{-\sigma_c}$ is the marginal utility of consumption at date t.

Labour supply and wage setting

There is monopolistic competition in the labor market, different types of labor are supplied by different households. Wages are set by unions representing the interest of households supplying the same type of labour. Union i sets $W_t(i)$, the nominal wage level belonging to type i of labor. The composite labor good of the economy is a CES aggregate of different types of labor,

$$L_t = \left(\int_0^1 l_t(i)^{\frac{\theta_t^w - 1}{\theta_t^w}} di\right)^{\frac{\theta_t^w}{\theta_t^w - 1}}, \tag{4}$$

where $\theta_t^w > 1$ is elasticity of substitution between different types of labor, $\theta_t^w = \theta^w \tilde{\theta}_t^w$, where $\theta^w > 1$ is a given parameter, and $\tilde{\theta}_t^w$ is governed by an exogenous AR1 process.

Equation (4) implies that the demand for labor supplied by household i is given by

$$L_t(i) = \left(\frac{W_t}{W_t(j)}\right)^{\theta_t^w} L_t, \tag{5}$$

where the aggregate wage index W_t is defined by

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta_t^w} dj\right)^{\frac{1}{1-\theta_t^w}}.$$

It is assumed that there is sticky wage setting in the model, as in the paper of Erceg et al. (2000).[2] Similarly to Calvo (1983), every union at a given date changes its wage in a rational, optimizing forward-looking manner with probability $1-\xi_w$. All those unions, which do not optimize at the given date follow a rule of thumb:

$$W_{t+1}(i) = W_t \Pi_t^{wind},$$

where $\Pi_t^{wind} = \Pi_t^{\gamma_w} \Pi^{1-\gamma_w}$, Π denotes the steady-state inflation and γ_w is the degree of wage indexation. Define $\Pi_{T,t}^{wind} = \Pi_T^{wind} \Pi_{T-1}^{wind} \dots \Pi_t^{wind}$ if $T > t$ and $\Pi_{t,t}^{wind} = 1$. Then the wage setting scheme of the non-optimizers at date $T \geq t$ is described by formula

$$W_T(i) = W_t(i) \Pi_{T,t}^{wind}. \tag{6}$$

If a union chooses its wage optimally at date t it has to take into account that it will follow the rule of thumb at $t+1$ with a probability of ξ_w, at $t+2$ with ξ_w^2, and so on. Hence it has to weight the objective function with the above sequence of probabilities. That is, it maximizes formula

$$\sum_{T=t}^{\infty} (\xi_w \beta)^{T-t} \mathrm{E}_t \left[\varepsilon_T^b \left\{ U(H_T, L_T(i)) - V(M_T) \right\} \right],$$

subject to the budget constraint (2), the labor demand equation (5) and the indexation scheme (6), furthermore, $H_t = C_t - hC_{t-1}$.

If formulas (5) and (6) are substituted into the objective then the first-order conditions with respect to consumption and the nominal wage are the following,

$$\bar{\lambda}_T = \frac{\Lambda_t}{P_T}, \tag{7}$$

$$\sum_{T=t}^{\infty} (\xi_w \beta)^{T-t} \mathrm{E}_t \left[L_T \left(\frac{W_T}{W_t(i) \Pi_{T,t}^{wind}} \right)^{\theta_t^w} \bar{\lambda}_T \Pi_{T,t}^{wind} \right] = \tag{8}$$

$$\frac{\theta_t^w}{\theta_t^w - 1} \sum_{T=t}^{\infty} (\xi_w \beta)^{T-t} \mathrm{E}_t \left[L_T \left(\frac{W_T}{W_t(i) \Pi_{T,t}^{wind}} \right)^{\theta_t^w} \varepsilon_T^b \frac{L_T(i)^{\sigma_l}}{W_t(i)} \right],$$

where $(\xi_w \beta)^{T-t} \bar{\lambda}_T$ is a state contingent lagrangian multiplier of the budget constraint (2) and $\Lambda_T = \varepsilon_T^b H_t^{-\sigma_c}$.

The Calvo assumption and equation (4) imply that the aggregate wage index is governed by the following equation.

$$W_t^{1-\theta_t^w} = (1 - \xi_W)(W_t^\star)^{1-\theta_t^w} + \xi_W \left(W_{t-1} \Pi_{t-1}^{wind} \right)^{1-\theta_t^w}, \tag{9}$$

where $W_t^\star$ is the nominal wage chosen by optimizing unions at date t.

Finally, note that instead of the shock $\tilde{\theta}_t^w$ its transformation will be estimated,

$$\log \varepsilon_t^w = (1 - \beta \xi_w)(1 - \xi_w) \left[(1 + \theta^w \sigma_l) \xi_w (\theta^w - 1) \right]^{-1} \log \tilde{\theta}_t^w,$$

furthermore

$$\log \varepsilon_t^w = \rho_w \log \varepsilon_{t-1}^w + u_t^w, \qquad \text{iid. } u_t^w \sim N\left(0, \sigma_w^2\right).$$

[2]See, furthermore, Kollmann (1997), CEE (2005). Most recent references are Adolfson et al. (2005) and Fernandez-Villaverde and Rubio-Ramirez (2006).

2.2 Production

Final goods production has a hierarchical structure: at the first stage labor and physical capital are transformed into differentiated intermediate inputs in a monopolistically competitive industry. At the second stage retailers produce a homogenous final good using the differentiated inputs in a perfectly competitive environment.

Entrepreneurs and capital producers are assumed to be distinct agents due to purely technical reasons: the model becomes much more tractable if investment decisions (belonging to capital producers) and borrowing decisions (belonging to entrepreneurs) are separated.

Final good production

Retailers produce final good Y_t in a competitive market by a constant-returns-to-scale technology from a continuum of differentiated intermediate goods $Y_t(j)$, $j \in [0,1]$. The technology is represented by the following CES production function:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta_t - 1}{\theta_t}} \, di \right)^{\frac{\theta_t}{\theta_t - 1}}, \tag{10}$$

where $\theta_t > 1$ measures the degree of the elasticity of substitution at date t, $\theta_t = \theta\tilde{\theta}_t$, where $\theta > 1$ is a parameter and $\tilde{\theta}_t$ is determined by an exogenous AR1 process.

The aggregate price index P_t is given by

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta_t} \, dj \right)^{\frac{1}{1-\theta_t}}, \tag{11}$$

where $P_t(j)$ denotes the prices of differentiated goods $Y_t(j)$, and the demand for $Y_t(j)$ is determined by

$$Y_t(j) = \left(\frac{P_t}{P_t(j)} \right)^{\theta_t} Y_t. \tag{12}$$

Intermediate good producer j can be described by a *Cobb-Douglas* technology,

$$Y_t(j) = A_t \left[u_t \overline{K}_t(j) \right]^{\alpha} \overline{L}_t(j)^{1-\alpha} - \mathsf{F},$$

where A_t is a common total factor productivity shock, u_t is the effective utilization rate of capital, $\overline{K}_t(j)$ and $\overline{L}_t(j)$ is the firm's demand for capital and labour, respectively. It is assumed that $A_t = A\varepsilon_t^a$, where A is the steady-state value of A_t and ε_t^a is and exogenous shock determined by the following AR1 process $\log \varepsilon_t^a = \rho_a \log \varepsilon_{t-1}^a + u_t^a$ with $u_t^a \sim N\left(0, \sigma_a^2\right)$. The parameter $0 < \alpha < 1$ and F represents fixed cost of the production process. Cost minimization implies the following input demand functions,

$$\overline{L}_t(j) = \frac{1-\alpha}{\mathsf{a}} \left(\frac{Z_t}{W_t^r} \right)^{\alpha} \frac{Y_t(j) + \mathsf{F}}{A_t}, \tag{13}$$

$$\overline{K}_t(j) u_t = \frac{\alpha}{\mathsf{a}} \left(\frac{W_t^r}{Z_t} \right)^{1-\alpha} \frac{Y_t(j) + \mathsf{F}}{A_t}. \tag{14}$$

where Z_t is the real rental rate of capital, $W_t^r = W_t/P_t$ denotes real wages and $\mathsf{a} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. The accompanying marginal cost function is

$$MC_t = \frac{(P_t Z_t)^{\alpha} W_t^{1-\alpha}}{A_t \mathsf{a}}. \tag{15}$$

Note that due to the constant returns-to-scale technology the marginal cost is uniform in the whole industry.

Let us consider how intermediate goods producers set their prices. It is assumed that prices are sticky: as in the model of Calvo (1983), each intermediate good producer at a given date changes its price in a rational, optimizing, forward-looking way with a constant probability of $1-\xi_p$. Those firms which do not optimize at the given date follow a rule of thumb. Rule of thumb price setters increase their prices by the expected average rate of inflation, as in Yun (1996), and to some extent by the difference between the past actual and perceived average inflation rates, similarly to Christiano et al. (2005) and Smets and Wouters (2003). Formally, if firm j does not optimize at date $t+1$

$$P_{t+1}(j) = P_t(j)\Pi_t^{ind},$$

where $\Pi_t^{ind} = \Pi^{\gamma_p}\Pi^{1-\gamma_p}$, Π is the steady-state inflation and γ_p measures the degree of indexation according to past inflation. Define $\Pi_{T,t}^{ind} = \Pi_T^{ind}\Pi_{T-1}^{ind}\cdots\Pi_t^{ind}$ if $T>t$ and $\Pi_{t,t}^{ind}=1$. Then if a given firm does not optimize after t its price at date $T \geq t$ is given by

$$P_T(i) = P_t(j)\Pi_{T,t}^{ind}. \tag{16}$$

If $P_t(j)$ is the chosen price of a firm at date t, then its profit will be at T

$$\begin{aligned} \digamma_T(P_t(j)) &= Y_T(j)\left(P_T(j) - MC_T\right). \\ &= Y_T P_T^{\theta_t}\left[\left(P_t(j)\Pi_{T,t}^I\right)^{1-\theta_t} - \left(P_t(j)\Pi_{T,t}^I\right)^{-\theta_t} MC_T\right]. \end{aligned}$$

The second equation is a consequence of formulas (12) and (16). If firm j sets its price optimally at date t it solves the following maximization problem.

$$\max_{P_t(j)} = \mathrm{E}_t\left[\sum_{T=t}^{\infty}(\xi_p\beta)^{T-t}\frac{\lambda_T}{\lambda_t}\digamma_T(P_t(j))\right],$$

where

$$\frac{\lambda_T}{\lambda_t} = \beta^{T-t}\frac{\Lambda_T/P_T}{\Lambda_t/P_t},$$

is the stochastic discount factor and recall that Λ_t is the marginal utility of consumption of optimizing consumers, who owns the firms. The first-order condition is

$$\sum_{T=t}^{\infty}(\xi_p\beta)^{T-t}\mathrm{E}_t\left[\frac{\Lambda_T}{P_T}Y_T\left(\frac{P_T}{P_t(i)\Pi_{T,t}^I}\right)^{\theta_t}\left(\Pi_{T,t}^I - \frac{\theta_t}{\theta_t-1}\frac{P_T mc_T}{P_t(i)}\right)\right] = 0,$$

where $mc_T = MC_T/P_T$ is the real marginal cost. The above condition implies that all firms choose the same $P_t(i)$. Let us denote this uniform price by $P_t^\star$. Rearranging the above expression yields

$$\frac{P_t^\star}{P_t} = \frac{\theta_t}{\theta_t-1}\frac{\sum_{T=t}^{\infty}(\xi_p\beta)^{T-t}\mathrm{E}_t\left[\Lambda_T Y_T\left(\frac{P_T}{P_t\Pi_{T,t}^{ind}}\right)^{\theta_t} mc_T\right]}{\sum_{T=t}^{\infty}(\xi_p\beta)^{T-t}\mathrm{E}_t\left[\Lambda_T Y_T\left(\frac{P_T}{P_t\Pi_{T,t}^{ind}}\right)^{\theta_t-1}\right]}. \tag{17}$$

Equation (10) and the price-setting assumptions imply that the evolution aggregate price index is given by

$$P_t^{1-\theta_t} = (1-\xi_p)\left(P_t^\star\right)^{1-\theta_t} + \xi_p\left(P_{t-1}\Pi_{t-1}^{ind}\right)^{1-\theta_t}. \tag{18}$$

It is important to note that instead of $\tilde{\theta}_t$ its transformation of will be estimated,

$$\log \varepsilon_t^p = (1 - \beta\xi_p)(1 - \xi_p)\left[\xi_p(\theta - 1)\right]^{-1} \log \tilde{\theta}_t,$$

and it is assumed that

$$\log \varepsilon_t^p = \rho_p \log \varepsilon_{t-1}^p + u_t^p, \qquad \text{iid. } u_t^p \sim N\left(0, \sigma_p^2\right).$$

Capital producers

It is assumed that capital goods are produced by a separate sector in a competitive market. Capital producers are price takers and owned by the representative households. At the end of date t they buy the depreciated physical capital stock $(1-\tau)K_t$ from the entrepreneurs and final goods I_t from retailers and convert them into capital stock K_{t+1} which is sold to entrepreneurs and used for production at date $t+1$. Capital production technology is described by the following equation

$$K_t = (1 - \tau) K_{t-1} + \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t x_t, \tag{19}$$

where Φ is the capital adjustment cost with properties $\Phi(1) = \Phi'(1) = 0$, $\phi \equiv \Phi''(1) > 0$ and x_t is an exogenous productivity shock. For technical reasons, a transformation of the above shock is estimated: $\log \varepsilon_t^i = (1 + \beta) \log x_t$, and it is governed by an AR1 process: $\log \varepsilon_t^i = \rho_i \log \varepsilon_{t-1}^i + u_t^i$ with iid. $u_t^i \sim N\left(0, \sigma_i^2\right)$.

The representative capital producer maximizes their future discounted profit stream:

$$\max \sum_{t=0}^{\infty} \mathrm{E}_0 \left[\beta^t \frac{\lambda_t}{\lambda_0} \left\{P_t^k \left(K_{t+1} - (1-\tau)K_t\right) - P_t I_t\right\}\right],$$

subject to the constraint (19), where P_t^k is the price of capital goods. Substituting the constraint into the objective function yields

$$\max \sum_{t=0}^{\infty} \mathrm{E}_0 \left[\beta^t \frac{\lambda_t P_t}{\lambda_0 P_0} \left\{Q_t \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t x_t - I_t\right\}\right],$$

where $Q_t = P_t^k (P_t)^{-1}$ is the relative price of capital goods. The accompanying first order condition is

$$1 = Q_t x_t \left[1 - \Phi\left(I_t^{gr}\right) - \Phi'\left(I_t^{gr}\right) I_t^{gr}\right] + \beta \mathrm{E}_t \left[Q_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} x_{t+1} \Phi'\left(I_{t+1}^{gr}\right) \left(I_{t+1}^{gr}\right)^2\right], \tag{20}$$

where $I_t^{gr} = I_t / I_{t-1}$.

Entrepreneurs

The output of capital goods producers cannot be used directly for production. The contributions of entrepreneurs is necessary to make physical capital suitable for intermediate goods producers. It is also assumed that capital utilization rate u_t is set by the entrepreneurs.

Entrepreneur l buys stock $K_{t+1}(l)$ from capital goods producers at the end of period t. It determines the capital utilization rate $u_{t+1}(l)$ and transform $K_{t+1}(l)$ into an appropriate form for production. During the transformation process the entrepreneur experiences an idiosyncratic shock: the purchased capital $K_{t+1}(l)$ becomes $\omega(l)K_{t+1}(l)$, where $\log \omega(l)$ has zero mean and variance σ^2. Then $\omega(l)K_{t+1}(l)$ is rented by intermediate goods producers in a competitive market, i.e., the real rental rate Z_{t+1} is given for both the entrepreneurs and the producers. Finally, at the end of date t the used and depreciated capital stock is sold to capital goods producers.[3]

The cost of adjustment of $u_{t+1}(l)$ is $\Psi\left(u_{t+1}(l)\right)\omega(l)K_{t+1}(l)$. Hence the optimal level of the capital utilization rate is given by

$$Z_{t+1} = \Psi'\left(u_{t+1}\right),$$

this implies that u_{t+1} is uniform for all entrepreneurs.[4] This implies that an individual entrepreneur's real rate of return of capital is $R^k_{t+1}(l) = \omega(l)R^k_{t+1}$, where

$$R^k_{t+1} = \frac{Z_{t+1}u_{t+1} - \Psi\left(u_{t+1}\right) + Q_{t+1}(1-\tau)}{Q_t}. \tag{21}$$

Since entrepreneurs are independent agents they must borrow from households in order to purchase the necessary capital. At the end of date t entrepreneur l borrows an amount $B^e_{t+1}(l)$, with

$$B^e_{t+1}(l) = Q_tK_{t+1}(l) - NW_{t+1}(l),$$

where $NW_{t+1}(l)$ is its available net worth. It is assumed that the financial intermediary cannot observe the realization of $\omega(l)$ unless it pays monitoring cost. As discussed in *Appendix A*, under these circumstances the optimal loan contract has the following form: the lender and the borrower agree a non-default loan rate $\overline{R}$ when the contract is signed at the end of date t. At the end of period $t+1$ when $\omega(l)R^k_{t+1}$ is realized the borrower pays back $\overline{R}B^e_{t+1}$ and keeps the rest of his revenue. However, if due to a bad realization of $\omega(l)$, i.e. if $\omega(l)$ is smaller than a certain threshold $\bar{\omega}$, the entrepreneur cannot pay back its obligation the lender pays the monitoring cost and gets to keep what it finds. It is assumed that the monitoring cost is equal to $0 < \mu < 1$ proportion of the realized gross payoff.

The optimal contract has two important implications. First, due to presence of asymmetric information and monitoring cost the entrepreneurs have to pay an external finance premium over the riskless real rate R_t. Second, the financial premium can be expressed in the following way,

$$S_t = S\left(\frac{NW_{t+1}}{Q_tK_{t+1}}\right), \qquad S'(\cdot) < 0 \tag{22}$$

where the financial spread (premium) is defined as

$$S_t \equiv \mathrm{E}_t\left[\frac{R^k_{t+1}}{R_t}\right]$$

Formula (22) synthesizes the idea underlying the financial accelerator: in equilibrium the external financial premium is negatively related to aggregate net worth of borrowers. The intuition is that firms with higher leverage (lower net wroth to capital ratio) will have a greater probability of defaulting and will therefore have to pay a higher premium. Since net worth is procyclical (because of the procyclicality of profits and asset prices), the external finance premium becomes countercyclical and amplifies business cycles through an accelerator effect on investments, production and spending.

Entrepreneurs are risk neutral and have a finite expected horizon for planning purposes. The probability that an entrepreneur will survive until the next period is ϑ_t. This assumption ensures that

[3]The original version of Bernanke et al. (1999) is slightly different from the one presented here. In their model entrepreneurs are responsible for intermediate goods production as well. Our exposition follows Christiano et al. (2008) since in this way it is easier to handle the determination of the capital utilization rate.

[4]Adjustment cost of capital utilization is represented by the following function, $\Psi(u_t) = u\psi\left[\exp\left(\frac{u_t-1}{\psi}\right) - 1\right]$, where u is the steady-state value of u_t.

entrepreneurs' net worth (the firm equity) will never be enough to fully finance the new capital acquisition. Every period $1-\vartheta_t$ entrepreneurs exit and they consume their residual equity. On the other hand, each period $1-\vartheta_t$ entrepreneurs are born, hence the population of entrepreneurs is constant. However, for technical reasons, it is necessary that entrepreneurs start with positive net worth in order to enable them to begin operations. Hence it is assumed that in each period entrepreneurs posses W_t^e endowment.

The aggregate entrepreneurial net worth at the end of date t is given by

$$NW_{t+1} = \vartheta_t V_t + W_t^e, \tag{23}$$

where the entrepreneurial equity V_t (i.e. the wealth accumulated by entrepreneurs from operating firms) is defined by

$$V_t = R_t^k Q_{t-1} K_t - \left(R_{t-1} + \frac{\mathcal{M}_t}{Q_{t-1}K_t - NW_t} \right) \left(Q_{t-1}K_t - NW_t \right), \tag{24}$$

and $\mathcal{M}_t$ is the expected monitoring cost. Equation 24 states that the entrepreneurial equity is equal to the return on capital minus financial cost of capital where $\mathcal{M}_t/(Q_{t-1}K_t - NW_t)$ represents the external finance premium. As mentioned entrepreneurial consumption is given by $C_t^e = (1-\vartheta_t^e)\,V_t$

Combining equations (23) and (24) yields the law of motion for NW_{t+1},

$$NW_{t+1} = \vartheta_t \left[R_t^k Q_{t-1}K_t - \left(R_{t-1} + \frac{\mathcal{M}_t}{Q_{t-1}K_t - NW_t} \right) \left(Q_{t-1}K_t - NW_t \right) \right] + W_t^e. \tag{25}$$

which is the second basic ingredient of the financial accelerator.

The time varying survival rate is determined by an AR1 process, $\vartheta_t = \vartheta \varepsilon_t^f$, where ϑ is the steady-state survival rate and $\log \varepsilon_t^f = \rho_f \log \varepsilon_{t-1}^f + u_t^f$ with iid. $u_t^f \sim N\left(0, \sigma_f^2\right)$. The shock ε_t^f can be interpreted as a financial shock since it has direct impact on the level of aggregate net worth and the financial spread.

2.3 The monetary authority and the government

The monetary authority sets independently the short-term interest rate R_t^n following the rule

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\zeta_r} \left[\left(\frac{\Pi_{t-1}}{\Pi}\right)^{\zeta_\pi} \left(\frac{\mathsf{Y}_t}{\mathsf{Y}}\right)^{\zeta_y} \right]^{1-\zeta_r} \left(\frac{\Pi_t}{\Pi_{t-1}}\right)^{\zeta_{\Delta\pi}} \left(\frac{\mathsf{Y}_t}{\mathsf{Y}_{t-1}}\right)^{\zeta_{\Delta y}} \varepsilon_t^r, \tag{26}$$

where variables without time indices represent their steady-state value, $\mathsf{Y}_t = Y_t/Y_t^*$ represents the output gap, Y_t^* is the flexible price output[5] and ε^r is the monetary-policy shocks, governed by the following AR1 process, $\log \varepsilon_t^r = \rho_r \log \varepsilon_{t-1}^r + u_t^e$, with iid. $u_t^r \sim N\left(0, \sigma_r^2\right)$.

On the other hand, it transfers its seignorage revenue to the government. The government's budget constraint is

$$P_t G_t = T_t + \bar{T}_t,$$

where T_t is nominal lump-sum tax revenue, $\bar{T}_t$ is the nominal revenue/burden related to τ_p, τ_w and G_t is the real government consumption. It is assumed that $G_t = G g_t$, where G is the steady-state level of G_t. As a technical matter, instead of g_t its transformation is estimated, $\varepsilon_t^g = G/Y g_t$, where Y is the steady-state level of Y_t, furthermore, $\log \varepsilon_t^g = \rho_g \log \varepsilon_{t-1}^g + u_t^g$ with iid. $u_t^g \sim N\left(0, \sigma_g^2\right)$.

[5]In the flexible-price version of the model, there are not financial frictions, markup shocks are removed and $\xi_p = \xi_w = 0$.

2.4 Equilibrium conditions

Good market equilibrium is defined by the following equation,

$$Y_t = C_t + C_t^e + I_t + G_t + K_t \Psi(u_t) + \mathcal{M}_t. \tag{27}$$

Equilibria in input markets are given by

$$\int_0^1 \overline{L}_t(j)\, dj = L_t, \tag{28}$$

$$\int_0^1 \overline{K}_t(j)\, dj = \int_0^1 \omega(l) K_t(l)\, dl = K_t. \tag{29}$$

Note that the condition $\int_0^1 \omega(l)K_t(l)\, dl = K_t$ is stronger than simply assuming that the mean of ω is equal to 1. Here it is assumed that ω is distributed "evenly" among entrepreneurs with small and large capital demand.

2.5 Summary

The estimation of the model is based on the solution of its log-linear approximation reported in *Appendix B*. The estimation is implemented by software *Dynare*[6] applying the solution algorithm of Blanchard and Kahn (1980).

The log-linearised model is given by equations (39) – (53) in *Appendix C*. These equations are similar to the ones presented in the papers of SW. However, the connection between the rate of return on capital and the real interest rate is influenced by the leverage ratio, as indicated by formula (50). Furthermore, in the models of SW the level of net worth is irrelevant, while in the financial accelerator mechanism equation (52) is a key formula since it determines the dynamics of NW_t.

The above 15 log-linearised equations determine the evolution of 15 endogenous variables, such as the real GDP (Y_t), real consumption (C_t), real investments (I_t), capital stock (K_t), labour hours (L_t), aggregate net worth (L_t), real wages (W_t^r), real rental rate of capital (Z_t), real marginal cost (mc_t), relative price of capital (Q_t), real interest rate (R_t), real rate of return of capital (R_t^k), financial spread (S_t), nominal interest rate (R_t^n) and inflation rate (Π_t).

The model is driven by 8 structural shocks, such as the financial shock (ε^f), preference shock (ε_t^c), government-spending shock (ε_t^g), investments shock (ε_t^i), productivity shock (ε_t^a), price- and wage-markup shocks (ε_t^p, ε_t^w) and the monetary-policy shock (ε_t^r).

3 Bayesian estimation

3.1 Data and methodology

In order to estimate the model presented in *section 2*, we use data over the period 1980:1 – 2008:4 on financial and macroeconomic variables. The macroeconomic time series, i.e., real consumption, real investment, real GDP, employment,[7] real wages, the GDP deflator inflation and the nominal interest rate are constructed in Fagan et al. (2001). The financial variable used in the estimation process is the

[6] On Dynare, see Juillard (2004).

[7] Since there is no reliable aggregate measure of hours worked in the euro area we substitute it by employment data. Hence the model is complemented by equation (54), presented in *Appendix C*, in order to determine the behaviour of employment. See Adolfson et al. (2007) for an explanation of the formula.

corporate bond spread. Since there is no available euro-area measure of this variable for the entire time period, the spread between the German corporate bonds and the three(-year German government bond yields is employed.[8]

The financial spread measure is demeaned, the real macroeconomic series are detrended by the HP filter, $\lambda = 1600$, while inflation and the nominal interest rate are detrended by the same relatively smooth trend, $\lambda = 40,000$.

To estimate the models, we apply a likelihood-based Bayesian method described in An and Schorfheide (2005). The first step is to construct the likelihood function. This needs the reduced form rational-expectation solution of the model. Then one has to transform it into state-space form, and formulate the Kalman filter for calculating the likelihood function. In the next step, the likelihood function is combined with prior distributions in order to derive the posterior density function of parameters. Then one has to find numerically the mode of the posterior density function. Finally, the random-walk Metropolis-Hastings (MH) algorithm is used to generate the posterior distribution. The applied MH algorithm is based on 350,000 draws (2 parallel chains of 205,000 draws discarding the initial burn-in period of 30,000 iterations). To monitor the convergence of the MH algorithm the method of Brooks and Gelman (1998) is applied. In order to compare different model versions, the marginal likelihoods of models are calculated by the modified harmonic mean algorithm of Geweke (1998).[9]

3.2 Specifying prior distributions

Some parameters are not estimated but kept fixed from the start of the procedure. The discount factor β is set to be 0.99, the production factor parameter $\alpha = 0.3$, and the goods and labor market substitution parameters are $\theta = 6$ and $\theta_w = 3$. The capital depreciation rate is assumed to be $\tau = 0.025$, while the steady-state ratio of consumption to output is chosen to be 0.6. These values are common in business-cycle literature.

Prior distributions of estimated parameters are chosen similarly to Smets and Wouters (2003). All the variances of the shocks are assumed to be distributed as *inverted Gamma* distributions with a degree of freedom equal to 2, covering a large positive domain. Due to structural restrictions of the theoretical model the values of some parameters, such as autoregressive parameters of shocks, Calvo, indexation, habit formation and interest-rate smoothing parameters, have to be located in the range between 0 and 1. Their priors follow *Beta* distributions guaranteeing the required interval for the estimated values. Prior distributions of the rest of the parameters are *Normal*.

3.3 Inference for the parameters of the financial accelerator

As mentioned in *section 2.2*, equations (22) and (25) represent the financial accelerator mechanism. Their log-linear versions[10] are given by

$$\widehat{S}_t = -\varkappa \left(\widehat{NW}_{t+1} - \widehat{Q}_t - \widehat{K}_{t+1} \right), \tag{30}$$

$$\widehat{NW}_{t+1} = \beta^{-1}\vartheta \left[k \left(\widehat{R}^k_t - \widehat{R}_{t-1} \right) + k\,(S-1) \left(\widehat{Q}_{t-1} + \widehat{K}_t \right) + \left(\widehat{R}_{t-1} + \widehat{NW}_t \right) \right] + \widehat{\varepsilon}^f_t, \tag{31}$$

[8] We used the difference of the series WU0022 and WZ9812 published on the Bundesbank's webpage. The last part of the constructed series displays a very high correlation (85 per cent) with the same measure for the euro area. Hence, it is a reasonably good proxy for the whole sample period.

[9] For the numerical implementation of the estimation procedure we used software, Dynare, see Juillard (2004).

[10] It is assumed W^e_t and $\mathcal{M}_t$ are negligible.

where the *hat* denotes the log-deviation of a variable from its steady-state value and variables without time subscripts represents their steady-state values, $k = QK/NW$ is the steady-state inverse leverage ratio.

As discussed in *Appendix A* and B, the values of ϑ, k and $\varkappa$ cannot be chosen independently, they are determined by the optimal financial contract between entrepreneurs and financial intermediaries. Namely, for given steady-state spread (S), riskiness (σ) and monitoring cost (μ) parameters the solution of the optimal contract determines the value of k, and the average default rate $F_\sigma(\overline{\omega})$, where F_σ is the cumulative distribution function of ω representing a log-normal distribution with mean zero and variance $\exp(\sigma^2) - 1$. Furthermore, the coefficients $\vartheta(\beta, S, k)$ and $\varkappa(S, k, \overline{\omega})$ are functions of the above parameters as well as the solutions of the optimal contract.

The parameters μ and σ do not have clear empirical counterparts. On the other hand, the values of k, $F_\sigma(\overline{\omega})$ and S are observable: the long-run average of the leverage ratio is equal to 2, and the average default rate is equal 3%, while the average annualized value of our measure for the corporate bond spread is equal to 105.2 basis points.

As a consequence, if one wants to match the above empirical values then she has to choose appropriately the values of σ and μ. However, if these empirical values are reproduced then the values of ϑ and $\varkappa$ are already determined and cannot be set without any restrictions anymore. That is, if one takes into account the structural restrictions of the financial accelerator model and the available empirical steady-state values then the coefficients of equations (30) and (31) cannot be estimated freely.

If someone estimates the coefficients of the above equation by using time-series information and neglecting the structural restrictions then the estimated values of the parameters can be very different from the values based on structural consideration and observed steady-state variables. In fact, comparing the coefficients derived in the above two different ways provide a test for the validity of the structural restrictions of the financial accelerator model. If the two approaches provide similar coefficients then the restrictions of the model are verified by the data. On the other hand, if they are significantly different then the model imposes too strong restrictions, their validity are rejected by the data.[11]

This is the way how this paper investigates the validity of structural restrictions of the financial accelerator model. We estimate the model in three different ways and compare the results. The following subsections explain the details of the estimation process.

***Version A*: All structural restrictions are imposed**

In this version all the structural implications of the financial accelerator model are taken into account. In order to match the empirical steady-state values of the financial premium, the inverse leverage ratio and the failure rate (recall that $S = 1.00263$, quarterly measure, $k =$ and $F_\sigma(\overline{\omega}) = 0.03$) we choose appropriate value for σ and μ (0.1139 and 0.0223, respectively). However, setting these parameters perfectly determines ϑ and $\varkappa$, they are equal to 0.1081 and 0.9848, respectively.

Hence, $\varkappa$ and ϑ are set prior to the estimation process, that is, we do not utilize any time-series information for deriving their values.

***Version B*: The steady-state failure rate is ignored**

In this version in order to obtain some degrees of freedom to estimate $\varkappa$, we ignore the restriction imposed by the observed failure rate.

[11] In principle this would be a general approach to test structural restrictions of DSGE models. E.g, the parameter β is usually determined by steady-state considerations, however, since it also appears in the log-linearised dynamic model equations its value can be estimated by using time-series information.

In the estimation process we set S and k as previously, hence $\vartheta(\beta, S, k)$ is determined. On the other hand, $\varkappa$ is estimated directly by using time-series information jointly with other parameters of the model. After obtaining the estimated value of $\varkappa^*$, we have to find values for σ, μ and $\overline{\omega}$ consistent with the estimated coefficient. That is the equation $\varkappa^* = \varkappa(S, k, \overline{\omega})$ has to be solved, where $\overline{\omega}$ is a function of σ and μ. Note that the vector (σ, μ) cannot be chosen uniquely in this case. However, their particular values are irrelevant, since they do not play any role in other blocks of the model.

Practically, we estimate an auxiliary parameter $\bar{\varkappa}$, and $\varkappa = 0.0125 + 0.2975\bar{\varkappa}$. Furthermore, a *Beta* prior is imposed on $\bar{\varkappa}$.

Version C: The steady-state failure rate and leverage ratio are ignored

In *version B* the coefficient of equation (30) is estimated, but the coefficients of equation (31) are fully determined by steady-state considerations. Hence, here we introduce more freedom in deriving k and neglect its observable steady-state value.

In this version the values of S and μ are fixed (1.00263 and 0.0223, respectively) and we estimate σ^2 jointly with other parameters. The estimated value of σ^2 determines k, $F_\sigma(\overline{\omega})$, $\vartheta(\beta, S, k)$ and $\varkappa(S, k, \overline{\omega})$.

Practically, an auxiliary parameter $\bar{\sigma}^2$ with a *Beta* prior distribution is estimated. Then $\sigma^2 = 0.05 + 5.95\bar{\sigma}^2$.

Estimation of the pure SW model

As a benchmark, we also estimate the pure Smets – Wouters model. In this case equations (30) and (31) are ignored and it is assumed that the financial premium is fully explained by a financial shocks, that is, $\widehat{S}_t = \widehat{\varepsilon}_t^f$.

4 Estimation results

4.1 Validity of structural restrictions

As discussed in the previous section, we estimate in three distinct ways the Smets and Wouters model complemented with the financial accelerator mechanism (SWFA), and compare the outcomes of the different approaches in order to investigate the validity of structural restrictions imposed by the financial accelerator block.

Tables 5 – 7 display the estimation results. Comparing them reveals that the three approaches provide highly different values for the parameter $\varkappa$. While it is equal to 0.108 according to *version A*, *version B* delivers much lower values, e.g, the mode and mean values of $\varkappa$ are equal to 0.039 and x.zsw, respectively. On the other hand, *version C* provides much higher values, the estimated mode and mean of $\varkappa$ are equal to 0.3026 and 0.2675, respectively. Furthermore, in *version C* provide $k = 1.0053$ which is very different from its observable long run average value of 2.

It is interesting to note that the estimated magnitudes of $\varkappa$ in *versions A* and *B* is relatively close to the ones reported in other studies (see our *literature review* at and of this section), while it is much higher in *version C*. This is due to the fact that in *version C* the coefficients of equation (31) is quite low since the estimated value of k is nearly half of the steady-state value used in other cases.

In summary, the theoretical financial accelerator model imposes too strong structural restrictions which are not supported by the data, since time-series information and the observed steady-state values imply highly different values for the parameters under study.

4.2 Model comparison

We use *marginal likelihood* (or *data density*) measure for ranking the estimated model versions' empirical performance. The higher is the marginal likelihood of an estimated model the better it explains the data.

Comparing the results presented in *Tables 5 – 7* reveals that the least restricted *version C* has the best fit, while *version A* with the complete set of structural restrictions explains worst the data.

For more accurate evaluation of the relative performance of the model versions we use the *Bayes factor*, the ratio of marginal likelihoods,[12] Jeffrey (1961) suggested the rule presented in *Table 1* to interpret the Bayes factor.

According to *Table 1*, *Version C* is not simply better then the others but the evidences are *decisive* against *versions A* and *B*. Furthermore, there is also decisive evidence against *version A* compared to *version B*.

These results reinforce our findings on the validity of structural restrictions discussed in the previous subsection. Comparing the estimated parameters demonstrated the data did not support the structural restrictions of the model. However, if the 3 model versions described the data almost equally then one would argue that the above statement is irrelevant. But the model comparison based on the Bayes factor proves that this is not the case: the 3 versions' abilities to describe the data are significantly different.

It is also an important question whether the Smets – Wouters model combined with the financial accelerator (SWFA) can describe the data better than the pure SW model. Calculating the Bayes factors based using the results presented in *Tables 4 – 7* reveals that the SW model is definitely better than the *versions A* and *B* of the SWFA model: the evidences are decisive against them. Only *Version C* of the SWFA model beats the pure SW model: there is *strong* evidence against the SW model. That is, adding the financial accelerator mechanism can improve the pure SW model's ability to explain the data only if important numerical implications of the theoretical structure is released.

A possible interpretation of the above finding is that the qualitative features of the financial accelerator mechanism has the potential to improve standard DSGE models without financial frictions, however, its structural restrictions are too strong to apply their numerical implications. Probably, the problem with the structural restrictions are related to the restrictive assumptions introduced in BGG for the sake of tractability.

Table 1 Bayes factor decision rule

$B_{ij} < 1$	support for M_j
$1 < B_{ij} < 3$	very slight evidence against M_j
$3 < B_{ij} < 10$	slight evidence against M_j
$10 < B_{ij} < 100$	strong evidence against M_j
$B_{ij} > 100$	decisive evidence against M_j

4.3 Comparing our findings to previous empirical results

THIS SUBSECTION IS UNDER REVISION

Empirical papers studying the empirical performance of the financial accelerator can be classified them into two groups. The first includes the papers in which the estimation is done without the use of financial variables. The other one is composed by papers using financial variables in the estimation.

One of the main reference included in the first group is De Graeve (2008). He estimates for the U.S. *the same model we estimate*. He shows that there is a gain in fitting key macroeconomic aggregates by

[12] See Kass and Raftery (1995) for technical details.

including financial frictions in the model, i.e. the comparison of the SWFA with the pure SW shows that the former beats the latter.

Concerning the estimation strategy, the author first writes the model in a way such that it allows him to discard the bankruptcy cost and the threshold idiosyncratic shock from the model, avoiding to make assumptions about the distribution of idiosyncratic productivity shocks, as well as its parameters. As for the other financial parameters, he estimates all of them, assuming unform distributions for the capital to net worth ratio, the probability of survival and $\varkappa$ with bounds 0-5, 0.8-1 and 0-0.5 respectively obtaining posterior modes equal to 1.42, 0.9923 and 0.1005. He estimates also the steady state return on capital assuming a normal distribution with mean 1.015 and standard deviation 0.002 (equivalent to assume a normal distribution of the steady state risk premium with mean equal to 200 basis point) obtaining a posterior mode of 1.0131.

De Graeve (2008) results are in a large extent confirmed by Gelain (2009) for the Euro Area estimating *the same model* with data from 1980 to 2008 with no financial data, although he only estimates $\varkappa$ and ϑ fixing the other parameters to have empirically significant steady state ratios. Again the pure SWFA model performs better than the pure SW model. The point estimates are 0.0267 and 0.9797 assuming the same BGG calibration for the remaining financial parameters.

Queijo (2005, 2008) focuses on the Euro Area and the US. Estimating a *slightly different but comparable model* with the purpose of comparing the empirical relevance of the financial frictions in the Euro Area and in the US. Using data from 1980 to 2004 with no proxies for the premium, her findings confirm that a model featuring the accelerator framework fits data better. The pure SW is still beaten by the SWFA model.

She calibrates part of the parameters pertaining the financial sector as in BGG and she estimates using Bayesian techniques μ, ϑ, and $S-1$ (quarterly). The posterior modes are 0.117, 0.991, and 0.006 for the Euro Area and 0.083, 0.985, and 0.004 for U.S.. Those values imply a $\varkappa$ of about 0.04.

Similar results in Christensen and Dib (2008) for Canada: Using a maximum-likelihood procedure, they estimated a *simplified version* of the SW model augmented with the BGG framework and they find an improvement in the model's fit with the data when the financial accelerator is active. They only estimate $\varkappa$ at 0.042 calibrating all the other parameters. In a similar way Lopez and Rodriguez (2008) still find a higher likelihood for their model with financial frictions using Colombian data from 1980 to 2005 for the standard macroeconomic variables.

In what concerns Korea, Elekdag et al. (2005) find evidence of a sizable risk premium, and they conclude that this indicates that the financial accelerator mechanism is supported by the data. They do not compare their financial frictions featured version of the model with the pure SW model. They do not specify which calibration they use for the not estimated financial parameters.[13]

Gelain and Kulikov (2009) is still in his very preliminary version, but from their estimation of a small open economy model for Estonia based on *the same model* of Gelain (2009), adapted to describe the open economy aspects, the ability of the model with financial frictions to replicate better the data is very sensible to the specification of the model which represents "the rest of the world". In any case the accelerator mechanism works as in De Graeve (2008). The relevant estimated parameters $\varkappa$ and ϑ are 0.13 and 0.8 respectively.

Three papers belong to the second group. The first is Meier and Müller (2005), their estimation is done by minimizing the distance between the impulse response functions of a VAR and those of the DSGE model.

The only parameter they estimate is $\varkappa$. They fix those deep parameters determining it consistently

[13] Presumably They estimate $\varkappa$ and K/N obtaining posterior modes equal to 0.069 and 1.63. The implied median risk premium they get is 3.27 percent per quarter. This implies an annualized endogenous risk premium of about 13 percent.

with the desired net worth to capital ratio, determined in turn by $\varkappa$. They fix σ^2 at 0.27 and they allow for variations in μ between 0 and 0.4, and $(1-\vartheta)$ between 0.012 and 0.03 which imply simultaneous and proportionate variations in $\varkappa$ between 0 and nearly 0.1 (corresponding to no financial frictions and strong frictions respectively).

They obtain sizeable points estimates for the relevant parameter governing the financial sector ($\varkappa \cong 0.06$), but those fail to be statistically significant. The same conclusion is suggested in their analysis by their distance metric tests (very similar in spirit to likelihood ratio tests), which show financial frictions to have only a marginal impact on improving the model's fit with the data. Hence their version of the pure Smets and Wouters does not seem to make a bad job in describing the data if compared with its financial frictions featured version.

The second paper is Gilchrist et al. (2009). They estimate the Smets and Wouters 2007 model augmented with the BGG framework for the U.S. using Bayesian techniques. They introduce among the observables a series for the risk premium built as a medium-risk and long-maturity corporate bond spread. They estimate only $\varkappa$ at about 0.04 (with a beta prior with mean 0.07 and standard deviation 0.02). A comparison with the pure SW model is neither provided nor discussed in terms of data fit.

The third is Christiano et al. (2008). They use the model with financial frictions not with the purpose of comparing its properties with those of a model without those frictions, hence they do no provide any support in this respect. Nevertheless, they use a large set of non standard macroeconomic aggregates in the estimation. They do not estimate the financial sector parameters (with the exception of the shock related to that sector), but they calibrate them to have steady state ratios in line with the empirical ones. In particular, the values they give to the calibrated parameters are 97.8, 0.1, 2.60%, 0.12 for the Euro Area and 97.62, 0.33, 1.30%, 0.67 for the U.S. for ϑ, μ, $F_\sigma(\overline{\omega})$, and σ respectively.

Table 2 Comparison of previous estimations

	$\varkappa$	ϑ	S	K/N
BGG - U.S.	0.05 (c)	0.9728 (c)	1.02 (c)	2 (c)
De Graeve (2008) - U.S.	0.1005 (e)	0.9923 (e)	1.02 (i)	1.42 (e)
Gelain (2009) - Euro Area	0.0267 (e)	0.9797 (e)	1.02 (c)	2 (c)
Queijo (2008) - Euro Area	0.04 (i)	0.9910 (e)	1.024 (e)	2 (c)
Queijo (2008) - US	0.04 (i)	0.9850 (e)	0.083 (e)	2 (c)
Christensen and Dib (2008) - Canada	0.042 (e)	0.9728 (c)	1.0075 (c)	2 (c)
Lopez and Rodriguez (2008) - Colombia	0.059 (e)	0.9728 (c)	1.02 (c)	2 (c)
Elekdag et al. (2005) - Korea	0.069 (e)	?	1.13 (i)	1.63 (e)
Gelain and Kulikov (2009) - Estonia	0.13 (e)	0.8 (e)	2.81 (c)	
Meier and Müller (2005) - U.S.	0.06 (e)	0.970 - 0.988 (c)	0 - 1.04 (c)	2 (c)
Gilchrist et al. (2009) - U.S.	0.04 (e)	0.990 (c)	?	1.7 (c)
Christiano et al. (2008) - Euro Area	0.04 (c)	97.80 (c)	1.04 (c)	1.92 (c)
Christiano et al. (2008) - US	0.04 (c)	97.62 (c)	1.05 (c)	1.13(c)

(c) calibrated, (e) estimated, (i) implied by estimated parameters

5 Properties of the estimated model

This section reviews some properties of the estimated models by the tools of *variance decomposition* and *impulse response* analysis. We compare the pure SW model only to *version C* of the SWFA model since the other two versions described the data much worst than the benchmark model without any financial frictions.

5.1 Variance decomposition

We want to investigate two issues by the variance decomposition. First, we analyze the ability of the financial accelerator mechanism to explain endogenously the observed fluctuations of the financial premium. Second, we study the impact of the financial shock on key macroeconomic variables, we check whether it is an important shock from macroeconomic point of view. *Tables 8* and *9* summarize the results based on the estimated mode values.

As for the first issue, in the short run the SWFA model is very weak to explain the behaviour of the premium: 88.4 and 81.2 percent of its 1- and 4-quarter forecast errors are explained by the financial shock which directly influence this variable. In the long run the model performs better in this sense, but it is still moderately able to explain endogenously the premium: two thirds of its unconditional variance is explained by the financial shock.

Regarding the second issue, *Table 9* reveals that the impact of the financial shock is not isolated: beyond the financial premium it significantly influences the behaviour of investments and employment both in the short and the long run, and consumption and real wages in the long run. This is a distinctive characteristic of the SWFA model, in the pure SW model the impact of the financial shock is significantly smaller on these macroeconomic variables. In the presence of the financial accelerator, the financial shock replaces a significant part of the effects of the investments shock, and that of the consumption shock in a smaller degree.

5.2 Impulse response analysis

This section studies whether the financial accelerator mechanism works the same way as described in the original BGG model. In their model the FA mechanism has the following features. Adverse shocks results in the depreciation of entrepreneurs' net worth. As a consequence, the leverage increases and the financial spread increases. That is, the behaviour of the spreads is *counter-cyclical*. Due to the increasing premium financing investments will be more expensive and the decline of investments will be larger than without financial frictions. Hence, the decline of real activity is *enhanced* by the FA mechanism.

Financial shock

Figures 1a, 1b and 1c plot the responses to an adverse financial shock. The figures display the outcomes of the pure SW model and that of *version C* of the SWFA model. The magnitude of the shock are set in such away that the peaks of the rise of the premium are equal to 1 percent in both cases.

The effects of an adverse financial shock resemble that of adverse demand shocks. That is, the real GDP, investments, labor utilization decline, inflation, the nominal and the real interest rate decline. On the other hand, real consumption increases due to Ricardian representative households and the declining real interest rates.

It is surprising that the financial accelerator does not work in this case as one would expect. In the short run investments react similarly in both the SW and SWFA models, however, in the long run the decline of investments is deeper in the model without financial frictions.

Demand shocks

This subsection reviews the effects of demand shocks: namely, the preference, government-spending and investments shocks. In what follows, the magnitude of the hocks are set in such a way that the peak responses of the real GDP in the SWFA model are the same as in the presence of the discussed financial shock. Results are displayed in *Figures 2a – 4c*.

As mentioned, the main effects of demand shocks are similar to that of the financial shock: in all cases responses of the monetary policy, at least initially, are accomodative, real GDP and inflation decline.

Regarding the financial accelerator, financial imperfections enhance the decline of the real GDP in the presence of the preference shock, on the other hand, in the presence of the government-spending and investments shocks this feature is missing.

However, even in the case of the preference shock the enhancement of the real GDP decline works differently from the mechanism described in BGG. Despite the countercyclical behaviour of the premium, in the SWFA model investments increase and the magnified reaction of the real GDP is due to the large drop of consumption.

Although in the presence of the government-spending shock the premium is countercyclical as well, the drop of the real GDP is nearly the same in both model versions with and without financial frictions. This result is due to the fact that in the SWFA model a relatively large increase in consumption crowds out the effect of declining investments.

Finally, in the presence of investments shock the magnifying effect of financial imperfections do not work at all. The premium is procyclical, i.e., it decreases. The decline of the premium is due to the increase of the net worth. This happens because the decline of $\widehat{R}_t^k$ is smaller than that of $\widehat{R}_t$, see equation (31). As a consequence, the decline of investments is smaller in the SWFA model than in the pure SW model.

Monetary-policy shock

The effects of a monetary-policy shock is similar to that of demand shock, except for the behaviour of the interest rates, see *Figure 5a – 5c*.

In the presence of this shock, the decline of output is enhanced in the SWFA model. However, this is the consequence of the behaviour of consumption. Again, the financial spread is procyclical and the decline of investments is smaller in the model with financial frictions. The decrease of the spread is induced by a large drop of the relative price of physical capital (Q_t).

Supply shocks

This subsection studies impulse responses to the total-factor-productivity, the price-markup and the wage-markup shocks. The results are plotted in *Figures 6a – 8c*.

A common feature of adverse supply-side shocks is that they create inflationary pressure, hence, the monetary policy tries to neutralize this effect by tightening, although initial decline of the real interest rate is possible. The financial premium is procyclical all the three cases due to the drop of Q_t. In the cases of the productivity and price-markup shocks, the enlarged reaction of the real GDP in the SWFA model is caused by consumption. It is more surprising that in the presence of the wage-markup shock the decline of the real GDP is even reduced.

Evaluation

Table 3 summarizes results of the impulse response analysis. The table reveals that combining the SW model with the financial imperfections described in BGG does not provide the same mechanism as in the original financial accelerator model. In most cases the financial premium is procyclical, unlike in BGG. Furthermore, in half of the cases financial frictions do not enhance the reaction of output, and even if they do, it is due to the behaviour of consumption instead of investments.

We also checked whether the procyclicality of the premium is caused by the fact that in *version C* the coefficients are relatively small in equation (31) since k is nearly equal to 1. Increasing the value of k partly provides some remedies. If k is equal to 2 the premium becomes countercyclical in the presence of the policy-rule, productivity and the price-markup shocks. However, its procyclical behaviour remains robust for the magnitude of k in the presence of the investments and wage-markup shocks.

All in all, the problem is that the model version supported by the data hardly reproduces the financial accelerator mechanism described in BGG. In this sense other versions performs better, however, their data fit is very poor.

Table 3 Summary of the impulse-response analysis

Shocks	**Premium**	**Is the decline of output enhanced?**	**Is the decline of investments enhanced?**
Financial shock	countercyclical	no	no
Demand shocks			
Preference shock	countercyclical	yes	yes
Gov't-spending shock	countercyclical	no	yes
Investments shock	procyclical	no	no
Monetary-policy shock	procyclical	yes	no
Supply shocks			
Productivity shock	procyclical	yes	no
Price-markup shock	procyclical	yes	no
Wage-markup shock	procyclical	no	no

6 Conclusions

The objective of this paper is to investigate the empirical performance of the financial accelerator model presented in Bernanke et al. (1999, BGG) if it is combined with the DSGE model developed by Smets and Wouters (2003, 2005, 2007, SW). Using euro-area macroeconomic and financial data, we estimated the combined model (SWFA) by Bayesian methods and found that the particular form of the financial accelerator model we used has several serious shortcoming if it is taken to the data.

First, the theoretical financial accelerator model imposes structural restrictions not supported by the data. Removing some implications of these restrictions significantly alters the value of key estimated parameters and improves the ability of the model to explain the data.

Second, if all structural restrictions of the BGG model are applied then even the pure SW model without any financial frictions fits the data better. However, if some of the restrictions are removed then the empirical performance of the combined model is better than that of the SW model.

A possible interpretation of the above findings is that the qualitative features of the financial accelerator mechanism has the potential to improve standard DSGE models without financial frictions, however, the particular structural restrictions are too strong to apply its numerical implications. Probably, the problem with the structural restrictions are related to the restrictive assumptions introduced in BGG for the sake of tractability.

Third, although the theoretical financial accelerator model endogenously determines the external finance premium, the SWFA model is very weak in the short run, and moderate in the long run to explain empirical fluctuations of the premium. Most of the variance of the premium is explained by an exogenous shock. On the other hand, the financial shock replaces a significant part of the effects of investments and consumption shocks.

Finally, the best estimated version of the SWFA model does not reproduce the financial accelerator mechanism described in BGG, that is, the enhanced response of investments to shock due to counter-cyclical behaviour of the financial premium. In the presence of most estimated shocks the above mechanism does not work. In most cases the premium is procyclical and/or the movements of the investments are not magnified.

A Appendix: The optimal financial contract

As discussed in *section 2.2*, entrepreneur l borrows at the end of date t an amount $B^e_{t+1}(l)$, given by

$$B_{t+1}(l) = Q_t K_{t+1}(l) - NW_{t+1}(l). \tag{32}$$

The lender can observe the entrepreneur's realized return, $\omega(l) R^k_{t+1}$, only if she pays $\mu\omega(l) R^k_{t+1} Q_t K_{t+1}(l)$ monitoring cost, where $0 < \mu < 1$ is a given parameter.

The loan contract is specified as follows: a non-default loan rate $\overline{R}$ and a threshold value of the idiosyncratic shock $\omega(l)$, denoted $\overline{\omega}$, are defined in such a way that if the values of the idiosyncratic shock greater than or equal to $\overline{\omega}$, then the entrepreneur is able to repay the loan at the contractual rate. In other words, the entrepreneur l defaults if

$$\omega(l) < \overline{\omega} \equiv \frac{\overline{R} B^e_{t+1}(l)}{R^k_{t+1} Q_t K_{t+1}(l)}. \tag{33}$$

If default occurs the lender pays the auditing cost and keeps what it finds. That is, the intermediary's net receipts is $(1-\mu)\omega(l) R^k_{t+1} Q_t K_{t+1}(l)$. A defaulting entrepreneur receives nothing. On the other hand, if $\omega(l) > \overline{\omega}$, the entrepreneur repays the promised amount $\overline{R} B^e_{t+1}(l)$ and keeps the difference, equal to $\omega(l) R^k_{t+1} Q_t K_{t+1}(l) - R_t B^e_{t+1}(l)$.

The values of $\overline{\omega}$ and $\overline{R}_{t+1}(l)$ are determined by the requirement that the financial intermediary should receive an expected return equal to the opportunity cost of its funds. Because the loan risk in this case is perfectly diversifiable the relevant opportunity cost is the riskless rate, R_t. That is, the loan contract must satisfy

$$[1 - F_\sigma(\overline{\omega})]\, \overline{R} + (1-\mu) \int_0^{\overline{\omega}} \omega(l) R^k_{t+1} Q_t K^j_{t+1} dF_\sigma(\omega) = R_{t+1} B^e_{t+1}(l), \tag{34}$$

where F_σ is the log-normal cumulative density function of $\omega(l)$ with zero mean and $\exp(\sigma^2) - 1$ variance, furthermore f_σ denotes the accompanying probability distribution function.[14] Note that the probability of default is equal to $F_\sigma(\overline{\omega})$.

Combining equations (32), (33) and (34), thus eliminating $\overline{R}$, yields the participation constraint of the optimal contract problem, i.e.

$$\left\{ [1 - F_\sigma(\overline{\omega})]\, \overline{\omega} + (1-\mu) \int_0^{\overline{\omega}} \omega(l) dF_\sigma(\omega) \right\} R^k_{t+1} Q_t K_{t+1}(l) = R_t \left[NW_{t+1}(l) - Q_t K_{t+1}(l) \right]. \tag{35}$$

Let's define $\Gamma(\overline{\omega})$ and $1 - \Gamma(\overline{\omega})$ the fractions of net capital output received by the lender and the entrepreneur respectively, that is,

$$\Gamma(\overline{\omega}) \equiv \int_0^{\overline{\omega}} \omega(l) dF_\sigma(\omega) + \overline{\omega} \int_{\overline{\omega}}^{\infty} dF_\sigma(\omega)$$

The net share accruing to the lender is $\Gamma(\overline{\omega}) - \mu M(\overline{\omega}(l))$, where $\mu M(\overline{\omega}^j) \equiv \mu \int_0^{\overline{\omega}} \omega(l)\, dF_\sigma(\omega)$ is the expected monitoring cost, since the monitoring cost is equal to μ proportion of the realized gross payoff. Using the definitions of $\Gamma(\overline{\omega})$ and of $\mu M(\overline{\omega})$ equation (35) can be expressed as

$$\left[\Gamma(\overline{\omega}) - \mu M(\overline{\omega})\right] R^k_{t+1} Q_t K_{t+1}(l) = R_{t+1} \left[NW_{t+1}(l) - Q_t K_{t+1}(l) \right]. \tag{36}$$

[14] In our particular case,

$$f_\sigma(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{\left(\log x + \frac{1}{2}\sigma^2\right)^2}{\sigma^2}}.$$

The optimal contract is defined by the following problem,

$$\max_{\{\overline{\omega}\ K_{t+1}(l)\}} \left[1-\Gamma(\overline{\omega})\right] R^k_{t+1} Q_t K_{t+1}(l)$$

subject to equation (36). The first order conditions are given by,[15]

$$\begin{aligned} \Gamma'(\overline{\omega}) &= \Xi_t \left[\Gamma'(\overline{\omega}) - \mu M'(\overline{\omega})\right], \\ \Xi_t &= \frac{R^k_{t+1}}{R_t} \left\{\left[1-\Gamma(\overline{\omega})\right] + \Xi_t \left[\Gamma(\overline{\omega}) - \mu M(\overline{\omega})\right]\right\}, \end{aligned}$$

where Ξ_t is the Lagrangian multiplier. Combining the above two conditions yields

$$\mathrm{E}_t\left[R^k_{t+1}\right] = \varrho\left(\overline{\omega}\right) R_t, \tag{37}$$

where

$$\varrho\left(\overline{\omega}\right) = \left\{\frac{\left[1-\Gamma(\overline{\omega})\right]\left[\Gamma'(\overline{\omega}) - \mu M'(\overline{\omega})\right]}{\Gamma'(\overline{\omega})} + \left[\Gamma(\overline{\omega}^j) - \mu M(\overline{\omega}^j)\right]\right\}^{-1} \quad \text{with } \varrho\left(\overline{\omega}\right) > 1.$$

Define $S_t \equiv \mathrm{E}_t\left[R^k_{t+1}/R_t\right]$ as the external finance premium.[16] This ratio captures the premium of external finance over the safe interest rate due to the existence of monitoring costs. Equations (36) and (37) imply the following relationship between the aggregate capital expenditure $Q_t K_{t+1}$ and the aggregate net worth NW_{t+1},

$$Q_t K_{t+1} = \left\{\frac{1}{1 - S_t\left[\Gamma(\overline{\omega}) - \mu M(\overline{\omega})\right]}\right\} NW_{t+1} \tag{38}$$

Equation (38) is a key relationship in this context, for it explicitly shows the link between capital expenditure and entrepreneurs' financial conditions (summarized by aggregate net worth). One can view (38) as a demand equation, in which the demand for capital depends inversely on the price and positively on aggregate financial conditions. Equation (38) is expressed as equation (22) in *section 2.2*.

B Appendix: The steady state

Variables without time indices indicate their steady-state values.

B.1 Steady state of the variables in the SW model

The steady state of variables of the models of Smets – Wouters can be calculated independently from the financial-accelerator variables if the steady-state value of the financial spread is given.

We assume that $\Pi = 1$ and $A = 1$. Hence $R = R^n$. The Euler equation implies that $R = \beta^{-1}$ and equation (20) implies that $Q = 1$.

In the steady state, equation (21) can be written as $R^k = Z + (1-\tau)$. Since $R^k = SR = S\beta^{-1}$ one can calculate Z straightforwardly.

Using the formula defining the real marginal cost, one can express the steady-state value of the real wages as

$$W^r = \left(\frac{\mathsf{a} mc}{Z^\alpha}\right)^{\frac{1}{1-\alpha}}.$$

[15] See the *Appendix A* of BGG for further details.

[16] Given that $\varrho\left(\overline{\omega}\right) > 1$, S_t is necessary bigger than 1.

Define $\kappa = L/K$, then the first-order conditions of firms' cost minimization problem implies that

$$\kappa = \frac{1-\alpha}{\alpha}\frac{Z}{W^r}.$$

Define $\mathsf{I} = I/Y$, then the Cobb-Douglas production function and equation (19) imply that $\mathsf{I} = \tau\kappa^{\alpha-1}(1+\mathsf{f})$, where $\mathsf{f} = \mathsf{F}/Y$. Define $\mathsf{C} = C/Y$ and $\mathsf{G} = G/Y$. We take C as given. Then $\mathsf{G} = 1 - \mathsf{C} - \mathsf{I}$. Furthermore, define $\varpi = C/K$. Then using the Cobb-Douglas production function, equation (19) and the GDP identity, it is easy to show that

$$\varpi = \frac{1-\mathsf{G}}{1+\mathsf{f}}\kappa^{1-\alpha} - \tau.$$

The steady-state labour supply formula is given by

$$W^r = \frac{\theta^w}{\theta^w - 1}L^{\sigma_l}\left((1-h)C\right)^{\sigma_c}.$$

Substituting $L = \kappa K$ and $C = \varpi K$ into the above formula yields

$$K = \left\{\frac{\theta_w - 1}{\theta_w}W^r\left[(1-h)\varpi\right]^{-\sigma_c}\kappa^{-\sigma_l}\right\}^{\frac{1}{\sigma_c+\sigma_l}}.$$

Then it is straightforward to calculate C, L, Y, I, G.

B.2 Steady-state solution of the financial contract

The optimal loan contract described in *Appendix A* is linearly homogenous in K_t and NW_t. Hence for given σ, μ and S, its steady-state solution provides $\overline{\omega}$, $k = QK/NW = K/NW$ and Ξ. According to formula (38), the solution also establishes the following function,

$$S = \mathsf{k}(k),$$

it maps the steady-state inverse leverage ratio to the steady-state financial premium. As mentioned in *section 3.3*, the following dynamic log-linear equation is a key element of the financial accelerator mechanism,

$$\widehat{S}_t = -\varkappa\left(\widehat{NW}_{t+1} - \widehat{Q}_t - \widehat{K}_{t+1}\right).$$

It is obvious that

$$\varkappa = \frac{dS}{dk}\frac{k}{S}.$$

It is shown *Appendix A* of BGG (see pages 1386-87) that

$$\frac{dS}{dk} = \frac{S\left[\Gamma'(\overline{\omega} - \Xi\left(\Gamma'(\overline{\omega} - \mu M'(\overline{\omega})\right] - \Xi'\frac{\partial\overline{\omega}}{\partial S}\right.}{\Xi'\frac{\partial\overline{\omega}}{\partial k}}.$$

It is also demonstrated in BGG that the above formula possesses closed form solution (see pages 1381, 1383 and 1385) since

$$\begin{aligned}
\Xi' &= \frac{\mu\left[\Gamma'\left(\overline{\omega}\right)M''\left(\overline{\omega}\right) - \Gamma''\left(\overline{\omega}\right)M'\left(\overline{\omega}\right)\right]}{\left[\Gamma'\left(\overline{\omega}\right) - \mu M'\left(\overline{\omega}\right)\right]^2},\\
\frac{\partial\overline{\omega}}{\partial S} &= \frac{-\left[\Gamma\left(\overline{\omega}\right) - \mu M\left(\overline{\omega}\right)\right]}{\left[\Gamma'\left(\overline{\omega}\right) - \mu M'\left(\overline{\omega}\right)\right]S},\\
\frac{\partial\overline{\omega}}{\partial k} &= \frac{1}{\left[\Gamma'\left(\overline{\omega}\right) - \mu M'\left(\overline{\omega}\right)\right]S},
\end{aligned}$$

and

$$\Gamma(\overline{\omega}) = F^n(z-\sigma) - \overline{\omega}\left[1 - F^n(z)\right],$$
$$\Gamma(\overline{\omega}) - \mu M(\overline{\omega}) = (1-\mu)F^n(z-\sigma) - \overline{\omega}\left[1 - F^n(z)\right],$$

where F^n is the standard normal cumulative distribution function and $z = \left(\log \overline{\omega} + 0.5\sigma^2\right)/\sigma$. Furthermore,

$$\Gamma'(\overline{\omega}) = 1 - F_\sigma(\overline{\omega}), \qquad \Gamma''(\overline{\omega}) = -f_\sigma(\overline{\omega}),$$

and

$$M'(\overline{\omega}) = \overline{\omega} f_\sigma(\overline{\omega}), \qquad M''(\overline{\omega}) = f_\sigma(\overline{\omega})\left(-\frac{\log \overline{\omega}}{\sigma^2} - \frac{1}{2}\right).$$

All in all, for given σ, μ and S, the steady-state solution of the optimal contract problem determines the steady-state inverse leverage ratio (k), the average probability of default ($F_\sigma(\overline{\omega})$) and the elasticity of the financial premium with respect to the leverage ratio ($\varkappa$).

B.3 Steady state of the net worth

Setting k also determines the steady-state value of net worth since $NW = k^{-1}K$. However, the steady-state version of equation (25) also provides a constraint for the value of the net worth,

$$NW = \vartheta\left[\left(R^k - R\right)K + RNW\right],$$

where we assumed that C^e and $\mathcal{M}$ are negligible. This implies that the value of ϑ cannot be chosen freely since the steady-state values of all the variables in the above formula have already been determined. That is,

$$\vartheta = \frac{NW}{\left(R^k - R\right)K + RNW} = \frac{1}{R\left[1 + k(S-1)\right]}.$$

This is a natural consequence of the fact that the value of the endogenous variable S was chosen exogenously (based on its empirical steady-state value). If the value of an endogenous variable is determined exogenously then one has to adjust the value of one parameter accordingly to preserve the consistency of the model.

C Appendix: The log-linearized model

This section presents the log-linearised model. The *hat* denotes the log-deviation of a variable from its steady-state value and variables without time subscripts represents their steady-state values. Furthermore, $\pi_t = \hat{P}_t - \hat{P}_{t-1}$.

C.1 Aggregate demand

The path of aggregate consumption is described by the Euler equation (3), its log-linearised version is given by

$$\widehat{C}_t = \frac{1}{1+h}\mathrm{E}_t\left[\widehat{C}_{t+1}\right] + \frac{h}{1+h}\widehat{C}_{t-1} - \frac{1-h}{\sigma_c(1+h)}\widehat{R}_t + \widehat{\varepsilon}_t^c, \tag{39}$$

where

$$\widehat{\varepsilon}_t^c = \frac{1-h}{\sigma_c(1+h)}\widehat{\varepsilon}_t^b.$$

The real interest rate is given by the Fisher equation

$$\widehat{R}_t = \widehat{R}_t^n - \mathrm{E}_t\left[\pi_{t+1}\right]. \tag{40}$$

Log-linearising expression (21) yields[17]

$$\widehat{R}_t^k = \frac{Z\widehat{Z} + (1-\tau)\widehat{Q}_t}{1-\tau+Z} - \widehat{Q}_{t-1}. \tag{41}$$

The next expression is the GDP identity, derived from equation (27)

$$\widehat{Y}_t = \frac{C}{Y}\widehat{C}_t + \frac{I}{Y}\widehat{I}_t + \frac{K}{Y}Z\psi\widehat{Z}_t + \widehat{\varepsilon}_t^g, \tag{42}$$

where $\varepsilon_t^g = G/Y g_t$ and $\psi = \Psi'(1)/\Psi''(1)$, and we assumed that the magnitudes of C_t^e and $\mathcal{M}_t$ are negligible.

Log-linearising the first-order condition (20) yields the equation describing the investments dynamics,

$$\widehat{I}_t = \frac{\beta}{1+\beta}\mathrm{E}_t\left[\widehat{I}_{t+1}\right] + \frac{1}{1+\beta}\widehat{I}_{t-1} + \frac{1}{\varphi(1+\beta)}\widehat{Q}_t + \widehat{\varepsilon}_t^i, \tag{43}$$

where $\widehat{\varepsilon}_t^i = (1+\beta)\widehat{x}_t$ and the parameter $\varphi = \Phi''(1)$. Capital accumulation is determined by equation (19), that is,

$$\widehat{K}_t = \tau\left(\widehat{I}_t + \varphi\widehat{\varepsilon}_t^i\right) + (1-\tau)\,\widehat{K}_{t-1}. \tag{44}$$

C.2 Aggregate supply

Log-linearising and combining formulas (12), (13) and (28) yield the aggregate labor demand equation,

$$\widehat{L}_t = \alpha\left(\widehat{Z}_t - \widehat{W}_t^r\right) + \frac{\widehat{Y}_t}{1+\mathsf{f}} - \widehat{\varepsilon}_t^a, \tag{45}$$

where $\mathsf{f} = \mathsf{F}/Y$ and recall that $A_t = A\varepsilon_t^a$. Similarly, log-linearising and combining formulas (12), (14) and (29) yield the aggregate demand for physical capital,

$$\widehat{K}_t = (1-\alpha)\left(\widehat{W}_t^r - \widehat{Z}_t\right) - \psi\widehat{Z}_t + \frac{\widehat{Y}_t}{1+\mathsf{f}} - \widehat{\varepsilon}_t^a. \tag{46}$$

Using equation (15) one can derive the log-linear expression for real-marginal-cost dynamics

$$\widehat{mc}_t = \alpha\widehat{Z}_t + (1-\alpha)\,\widehat{W}_t^r - \widehat{\varepsilon}_t^a. \tag{47}$$

Log-linearising and combining equations (17) and (18) yield the hybrid New Keynesian Phillips curve,

$$(1+\beta\gamma_p)\pi_t = \beta\mathrm{E}_t\left[\pi_{t+1}\right] + \gamma_p\pi_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\widehat{mc}_t + \widehat{\varepsilon}_t^p, \tag{48}$$

[17] As mentioned, the adjustment cost of capital utilization is represented by the following function,

$$\Psi(u_t) = u\psi\left[\exp\left(\frac{u_t-1}{\psi}\right) - 1\right],$$

where u is the steady-state value of u_t. The given functional form implies $\Psi(1) = 0$, $\Psi'(1) = u$ and $\Psi''(1) = u/\psi$. The degree of capital utilization is determined by condition $\Psi'(u_t) = Z_t$. This implies that $u_t = \psi\log\left(\frac{Z_t}{Z}\right) + 1$ and $\Psi(u_t) = \psi\,(Z_t - Z)$. The above two expressions are used to replace variables u_t by Z_t.

where $\widehat{\varepsilon}_t^p = (1-\beta\xi_p)(1-\xi_p)\,\xi_p^{-1}(\theta-1)^{-1}\widehat{\widehat{\theta}}_t$. One can derive the wage Phillips curve by log-linearising and combining equations (7), (8) and (9),

$$\begin{aligned}(1+\beta)\widehat{W}_t &= \beta \mathrm{E}_t\left[\widehat{W}_{t+1}\right] + \widehat{W}_{t-1} + \beta \mathrm{E}_t\left[\pi_{t+1}\right] - (1+\beta\gamma_w)\,\pi_t + \gamma_w \pi_{t-1} \\ &+ \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\theta^w\sigma_l)\,\xi_w}\left[\frac{\sigma_c}{1-h}\left(\widehat{C}_t - h\widehat{C}_{t-1}\right) + \sigma_l \widehat{L}_t - \widehat{W}_t^r\right] + \widehat{\varepsilon}_t^w,\end{aligned} \tag{49}$$

where $\widehat{\varepsilon}_t^w = (1-\beta\xi_w)(1-\xi_w)\left[(1+\theta^w\sigma_l)\,\xi_w(\theta^w-1)\right]^{-1}\widehat{\widehat{\theta}}_t^w$.

C.3 Financial accelerator

The external finance premium (spread) is defined by

$$\widehat{S}_t = \mathrm{E}_t\left[\widehat{R}_{t+1}^k - \widehat{R}_t\right]. \tag{50}$$

As discussed, equation (22) implies that

$$\widehat{S}_t = -\varkappa\left(\widehat{NW}_{t+1} - \widehat{Q}_t - \widehat{K}_{t+1}\right). \tag{51}$$

The evolution of aggregate net worth is described by formula (25), if W_t^e and $\mathcal{M}_t$ are neglected it can be expressed as

$$\widehat{NW}_{t+1} = \vartheta\left[\frac{K}{NW}R\left(\widehat{R}_t^k - \widehat{R}_{t-1}\right) + \frac{K}{NW}R\,(S-1)\left(\widehat{Q}_{t-1} + \widehat{K}_t\right) + R\left(\widehat{R}_{t-1} + \widehat{NW}_t\right)\right] + \widehat{\varepsilon}_t^f. \tag{52}$$

C.4 Monetary policy

Log-linearising the policy rule (26) provides

$$\widehat{R}_t^n = \zeta_r \widehat{R}_{t-1}^n + (1-\zeta_r)\left(\zeta_\pi \pi_{t-1} + \zeta_y \widehat{\mathsf{Y}}_t\right) + \zeta_{\Delta\pi}\left(\pi_t - \pi_{t-1}\right) + \zeta_{\Delta y}\left(\widehat{\mathsf{Y}}_t - \widehat{\mathsf{Y}}_{t-1}\right) + \widehat{\varepsilon}_t^r. \tag{53}$$

C.5 Complementary employment equation

In order to estimate the model, we have to augment the above log-linear system with an *ad-hoc* employment equation,

$$\Delta\widehat{E}_t = \beta\Delta\widehat{E}_{t+1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{\xi_e}\left(\widehat{L}_t - \widehat{E}_t\right) \tag{54}$$

where $\Delta\widehat{E}_t = \widehat{E}_t - \widehat{E}_{t-1}$ and $\widehat{E}_t$ is the percentage deviation of the employment from its steady-state value. Furthermore, $0 < \xi_e < 1$.

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Table 4 Parameter estimates of the Smets – Wouters model

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Stand. err. of shocks							
financial	σ_f	I.Gam.	0.10	2*	0.102	0.104	[0.09, 0.12]
consumption	σ_c	I.Gam.	0.40	2*	0.110	0.143	[0.10, 0.19]
gov't. spending	σ_g	I.Gam.	0.30	2*	0.261	0.268	[0.24, 0.27]
investments	σ_i	I.Gam.	0.10	2*	1.022	0.930	[0.77, 1.08]
productivity	σ_a	I.Gam.	0.20	2*	0.594	0.664	[0.50, 0.80]
price markup	σ_p	I.Gam.	0.15	2*	0.1380	0.142	[0.12, 0.17]
wage markup	σ_w	I.Gam.	0.25	2*	0.172	0.177	[0.15, 0.21]
policy rule	σ_r	I.Gam.	0.10	2*	0.096	0.102	[0.09, 0.12]
Autoreg. coeff.							
financial	ρ_f	Beta	0.85	0.10	0.903	0.892	[0.85, 0.93]
consumption	ρ_c	Beta	0.85	0.10	0.714	0.557	[0.39, 0.73]
gov't. spending	ρ_g	Beta	0.85	0.10	0.803	0.797	[0.70, 0.90]
investments	ρ_i	Beta	0.85	0.10	0.438	0.433	[0.30, 0.55]
productivity	ρ_a	Beta	0.85	0.10	0.781	0.721	[0.59, 0.86]
price markup	ρ_p	Beta	0.50	0.10	0.370	0.328	[0.15, 0.51]
wage markup	ρ_w	Beta	0.50	0.10	0.280	0.283	[0.16, 0.39]
policy rule	ρ_r	Beta	0.85	0.10	0.583	0.591	[0.48, 0.69]
Utility function							
consumption	σ_c	Norm.	1.00	0.375	1.751	1.753	[1.23, 2.23]
habit	h	Beta	0.70	0.10	0.655	0.661	[0.51, 0.82]
labour	σ_l	Norm.	2.00	0.250	1.964	1.989	[1.56, 2.43]
Price setting							
Calvo prices	ξ_p	Beta	0.75	0.05	0.777	0.779	[0.74, 0.82]
Calvo wages	ξ_w	Beta	0.75	0.05	0.750	0.749	[0.69, 0.81]
ind. prices	γ_p	Beta	0.75	0.15	0.166	0.200	[0.07, 0.32]
ind. wages	γ_w	Beta	0.75	0.15	0.513	0.518	[0.29, 0.76]
Policy rule							
ir. smooth.	ζ_r	Beta	0.80	0.10	0.791	0.786	[0.73, 0.84]
inflation	ζ_π	Norm.	1.70	0.30	1.614	1.632	[1.25, 1.96]
outp. gap	ζ_y	Norm.	0.125	0.05	0.167	0.167	[0.09, 0.24]
change of inf.	$\zeta_{\Delta\pi}$	Norm.	0.30	0.10	0.203	0.210	[0.15, 0.27]
change of og.	$\zeta_{\Delta y}$	Norm.	0.0625	0.05	0.092	0.095	[0.05, 0.14]
*Other parameters***							
capacity util.	ψ	Norm.	0.20	0.005	0.174	0.172	[0.09, 0.25]
inv. adj. cost	ϕ	Norm.	4.00	1.50	6.400	6.408	[4.61, 8.11]
fixed cost	f	Norm.	0.20	0.05	0.219	0.213	[0.14, 0.29]
employment	ξ_e	Beta	0.50	0.15	0.713	0.712	[0.67, 0.75]
Log data density			-107.152				

* For the Inverted Gamma function the degrees of freedom are indicated.

** Recall, that $\psi = \Psi'(1)/\Psi''(1)$, $\phi = \Phi(1)$ and $\mathsf{f} = \mathsf{F}/Y$.

Table 5 Parameter estimates of *version A* of the SWFA model+

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Stand. err. of shocks							
financial	σ_f	I.Gam.	2.50	2*	0.691	0.693	[0.60, 0.80]
consumption	σ_c	I.Gam.	0.40	2*	0.182	0.163	[0.10, 0.21]
gov't. spending	σ_g	I.Gam.	0.30	2*	0.263	0.266	[0.24, 0.29]
investments	σ_i	I.Gam.	0.10	2*	1.089	1.103	[0.91, 1.33]
productivity	σ_a	I.Gam.	0.20	2*	0.771	0.780	[0.58, 1.00]
price markup	σ_p	I.Gam.	0.15	2*	0.136	0.141	[0.11, 0.17]
wage markup	σ_w	I.Gam.	0.25	2*	0.170	0.173	[0.14, 0.20]
policy rule	σ_r	I.Gam.	0.10	2*	0.117	0.122	[0.10, 0.14]
Autoreg. coeff.							
financial	ρ_f	Beta	0.50	0.10	0.496	0.484	[0.35, 0.61]
consumption	ρ_c	Beta	0.85	0.10	0.325	0.451	[0.24, 0.73]
gov't. spending	ρ_g	Beta	0.85	0.10	0.820	0.822	[0.72, 0.92]
investments	ρ_i	Beta	0.85	0.10	0.587	0.540	[0.39, 0.70]
productivity	ρ_a	Beta	0.85	0.10	0.618	0.618	[0.50, 0.73]
price markup	ρ_p	Beta	0.50	0.10	0.354	0.352	[0.19, 0.52]
wage markup	ρ_w	Beta	0.50	0.10	0.268	0.267	[0.17, 0.37]
policy rule	ρ_r	Beta	0.85	0.10	0.492	0.485	[0.39, 0.59]
Utility function							
consumption	σ_c	Norm.	1.00	0.375	1.743	1.850	[1.29, 2.29]
habit	h	Beta	0.70	0.10	0.812	0.701	[0.49, 0.87]
labour	σ_l	Norm.	2.00	0.250	1.958	1.927	[1.51, 2.35]
Price setting							
Calvo prices	ξ_p	Beta	0.75	0.05	0.790	0.787	[0.75, 0.83]
Calvo wages	ξ_w	Beta	0.75	0.05	0.782	0.770	[0.71, 0.84]
ind. prices	γ_p	Beta	0.75	0.15	0.171	0.194	[0.07, 0.31]
ind. wages	γ_w	Beta	0.75	0.15	0.533	0.514	[0.29, 0.73]
Policy rule							
ir. smooth.	ζ_r	Beta	0.80	0.10	0.734	0.729	[0.66, 0.80]
inflation	ζ_π	Norm.	1.70	0.30	1.391	1.458	[1.10, 1.81]
outp. gap	ζ_y	Norm.	0.125	0.05	0.194	0.189	[0.12, 0.26]
change of inf.	$\zeta_{\Delta\pi}$	Norm.	0.30	0.10	0.260	0.177	[0.20, 0.35]
change of og.	$\zeta_{\Delta y}$	Norm.	0.0625	0.05	0.088	0.094	[0.05, 0.14]
*Other parameters***							
capacity util.	ψ	Norm.	0.20	0.005	0.158	0.152	[0.07, 0.22]
inv. adj. cost	ϕ	Norm.	4.00	1.50	1.465	0.776	[0.78, 2.72]
fixed cost	f	Norm.	0.20	0.05	0.241	0.231	[0.15, 0.31]
employment	ξ_e	Beta	0.50	0.15	0.740	0.736	[0.70, 0.77]
Log data density			-144.790				

+ All structural implications of the *financial accelerator* model are taken into account.

$S = 1.00263$, $\mu = 0.2234$ $\sigma^2 = 0.1138$, $K/NW = 2$, $F_\sigma(\bar{\omega}) = 3\%$ are fixed prior to the estimation. These fixed values imply that $\varkappa = 0.108$ and $\vartheta = 0.848$.

* For the Inverted Gamma function the degrees of freedom are indicated.

** Recall, that $\psi = \Psi'(1)/\Psi''(1)$, $\phi = \Phi(1)$ and $\mathsf{f} = \mathsf{F}/Y$.

Table 6 Parameter estimates of *version B* of the SWFA model+

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Stand. err. of shocks							
financial	σ_f	I.Gam.	2.50	2*	2.003	2.057	[1.29, 2.73]
consumption	σ_c	I.Gam.	0.40	2*	0.185	0.178	[0.13, 0.22]
gov't. spending	σ_g	I.Gam.	0.30	2*	0.263	0.267	[0.23, 0.29]
investments	σ_i	I.Gam.	0.10	2*	0.956	0.970	[0.81, 1.13]
productivity	σ_a	I.Gam.	0.20	2*	0.722	0.756	[0.53, 0.96]
price markup	σ_p	I.Gam.	0.15	2*	0.134	0.139	[0.11, 0.17]
wage markup	σ_w	I.Gam.	0.25	2*	0.174	0.178	[0.15, 0.21]
policy rule	σ_r	I.Gam.	0.10	2*	0.105	0.110	[0.10, 0.13]
Autoreg. coeff.							
financial	ρ_f	Beta	0.50	0.10	0.382	0.376	[0.25, 0.51]
consumption	ρ_c	Beta	0.85	0.10	0.298	0.367	[0.19, 0.63]
gov't. spending	ρ_g	Beta	0.85	0.10	0.800	0.804	[0.70, 0.90]
investments	ρ_i	Beta	0.85	0.10	0.413	0.423	[0.30, 0.57]
productivity	ρ_a	Beta	0.85	0.10	0.647	0.645	[0.52, 0.77]
price markup	ρ_p	Beta	0.50	0.10	0.377	0.355	[0.17, 0.54]
wage markup	ρ_w	Beta	0.50	0.10	0.283	0.287	[0.17, 0.39]
policy rule	ρ_r	Beta	0.85	0.10	0.531	0.528	[0.41, 0.62]
*Financial accelerator***							
elasticity of S_t	$\varkappa$	Beta	0.50	0.287	0.039	0.039	[0.03, 0.05]
idiosync. risk	σ^2				0.347	0.347	[0.49, 0.26]
monit. cost	μ				0.006	0.006	[0.01, 0.01]
default rate	$F_\sigma(\bar{\omega})$				0.216	0.216	[0.31, 0.15]
Utility function							
consumption	σ_c	Norm.	1.00	0.375	1.737	1.837	[1.41, 2.43]
habit	h	Beta	0.70	0.10	0.860	0.801	[0.65, 0.92]
labour	σ_l	Norm.	2.00	0.250	1.987	1.977	[1.56, 2.35]
Price setting							
Calvo prices	ξ_p	Beta	0.75	0.05	0.779	0.781	[0.74, 0.82]
Calvo wages	ξ_w	Beta	0.75	0.05	0.761	0.755	[0.69, 0.81]
ind. prices	γ_p	Beta	0.75	0.15	0.168	0.204	[0.09, 0.33]
ind. wages	γ_w	Beta	0.75	0.15	0.602	0.573	[0.30, 0.79]
Policy rule							
ir. smooth.	ζ_r	Beta	0.80	0.10	0.784	0.780	[0.73, 0.83]
inflation	ζ_π	Norm.	1.70	0.30	1.590	1.655	[1.32, 2.00]
outp. gap	ζ_y	Norm.	0.125	0.05	0.239	0.135	[0.07, 0.20]
change of inf.	$\zeta_{\Delta\pi}$	Norm.	0.30	0.10	0.239	0.248	[0.19, 0.31]
change of og.	$\zeta_{\Delta y}$	Norm.	0.0625	0.05	0.049	0.055	[0.02, 0.10]
*Other parameters****							
capacity util.	ψ	Norm.	0.20	0.005	0.158	0.154	[0.09, 0.24]
inv. adj. cost	ϕ	Norm.	4.00	1.50	6.128	6.040	[4.26, 7.85]
fixed cost	f	Norm.	0.20	0.05	0.219	0.220	[0.14, 0.29]
employment	ξ_e	Beta	0.50	0.15	0.734	0.734	[0.70, 0.77]
Log data density			-122.423				

+ Some structural implications of the *financial accelerator* model are neglected. $S = 1.00263$, $k = K/NW = 2$ are fixed prior to the estimation, implying that $\vartheta = 0.9848$.

* For the Inverted Gamma function the degrees of freedom are indicated.

** Recall, that the *Beta* prior is imposed on $\bar{\varkappa}$, $\varkappa = 0.0125 + 0.2975\bar{\varkappa}$ and its estimated value determines ϑ, μ and $F_\sigma(\bar{\omega})$.

*** Recall, that $\psi = \Psi'(1)/\Psi''(1)$, $\phi = \Phi(1)$ and $\mathsf{f} = \mathsf{F}/Y$.

Table 7 Parameter estimates of *version C* of the SWFA model^{+}

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Stand. err. of shocks							
financial	σ_f	I.Gam.	1.00	2*	0.370	0.425	[0.30, 0.55]
consumption	σ_c	I.Gam.	0.40	2*	0.110	0.128	[0.09, 0.17]
gov't. spending	σ_g	I.Gam.	0.30	2*	0.261	0.262	[0.23, 0.28]
investments	σ_i	I.Gam.	0.10	2*	1.022	1.030	[0.88, 1.16]
productivity	σ_a	I.Gam.	0.20	2*	0.594	0.657	[0.50, 0.81]
price markup	σ_p	I.Gam.	0.15	2*	0.138	0.140	[0.11, 0.16]
wage markup	σ_w	I.Gam.	0.25	2*	0.172	0.176	[0.15, 0.20]
policy rule	σ_r	I.Gam.	0.10	2*	0.096	0.101	[0.09, 0.11]
Autoreg. coeff.							
financial	ρ_f	Beta	0.50	0.10	0.347	0.351	[0.25, 0.46]
consumption	ρ_c	Beta	0.85	0.10	0.714	0.628	[0.45, 0.81]
gov't. spending	ρ_g	Beta	0.85	0.10	0.803	0.797	[0.71, 0.87]
investments	ρ_i	Beta	0.85	0.10	0.438	0.423	[0.31, 0.56]
productivity	ρ_a	Beta	0.85	0.10	0.781	0.750	[0.61, 0.88]
price markup	ρ_p	Beta	0.50	0.10	0.370	0.353	[0.20, 0.54]
wage markup	ρ_w	Beta	0.50	0.10	0.280	0.286	[0.18, 0.39]
policy rule	ρ_r	Beta	0.85	0.10	0.583	0.586	[0.47, 0.70]
*Financial accelerator***							
idiosync. risk	$\bar{\sigma}^2$	Beta	0.50	0.287	0.908	0.862	[0.75, 0.99]
elasticity of S_t	$\varkappa$				0.303	0.268	[0.20, 0.37]
ss. leverage ratio	k				1.006	1.006	[1.01, 1.00]
replacement rate of entrp.	ϑ				0.989	0.988	[0.99, 0.99]
default rate	$F_\sigma(\bar{\omega})$				0.146	0.146	[0.13, 0.16]
Utility function							
consumption	σ_c	Norm.	1.00	0.375	1.874	1.778	[1.29, 2.26]
habit	h	Beta	0.70	0.10	0.561	0.639	[0.47, 0.81]
labour	σ_l	Norm.	2.00	0.250	1.870	1.912	[1.50, 2.36]
Price setting							
Calvo prices	ξ_p	Beta	0.75	0.05	0.776	0.777	[0.74, 0.82]
Calvo wages	ξ_w	Beta	0.75	0.05	0.751	0.750	[0.70, 0.81]
ind. prices	γ_p	Beta	0.75	0.15	0.165	0.201	[0.01, 0.30]
ind. wages	γ_w	Beta	0.75	0.15	0.500	0.526	[0.32, 0.80]
Policy rule							
ir. smooth.	ζ_r	Beta	0.80	0.10	0.811	0.803	[0.76, 0.86]
inflation	ζ_π	Norm.	1.70	0.30	1.670	1.708	[1.35, 2.05]
outp. gap	ζ_y	Norm.	0.125	0.05	0.158	0.158	[0.11, 0.22]
change of inf.	$\zeta_{\Delta\pi}$	Norm.	0.30	0.10	0.192	0.205	[0.15, 0.26]
change of og.	$\zeta_{\Delta y}$	Norm.	0.0625	0.05	0.094	0.090	[0.05, 0.14]
*Other parameters****							
capacity util.	ψ	Norm.	0.20	0.005	0.171	0.172	[0.10, 0.25]
inv. adj. cost	ϕ	Norm.	4.00	1.50	4.649	5.160	[3.77, 6.62]
fixed cost	f	Norm.	0.20	0.05	0.219	0.212	[0.15, 0.30]
employment	ξ_e	Beta	0.50	0.15	0.713	0.718	[0.68, 0.75]
Log data density			-103.836				

$^{+}$ Some structural implications of the *financial accelerator* model are neglected. $S = 1.00263$, $\mu = 0.02234$ are fixed prior to the estimation.

* For the Inverted Gamma function the degrees of freedom are indicated.

** Recall, that $\sigma^2 = 0.05 + 5.95\bar{\sigma}^2$ and its estimated value determines $\varkappa$, $k = K/NW$, ϑ and $F_\sigma(\bar{\omega})$.

*** Recall, that $\psi = \Psi'(1)/\Psi''(1)$, $\phi = \Phi(1)$ and $\mathsf{f} = \mathsf{F}/Y$.

Table 8 Variance decomposition of the Smets – Wouters model

	ε_f	ε_c	ε_g	ε_i	ε_a	ε_p	ε_w	ε_r
1-quarter forecast error decom.								
Financial premium	100.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0
Consumption	00.9	87.8	00.2	00.0	03.0	00.6	00.2	07.2
Investments	05.1	01.7	00.1	86.1	02.4	00.5	00.5	03.6
Output	00.5	29.4	28.5	30.4	01.5	01.3	00.2	08.3
Labour	00.1	08.6	08.4	08.9	71.1	00.2	00.2	02.4
Employment	01.5	13.3	08.6	14.9	46.9	01.3	02.6	11.0
Real wages	00.0	00.8	00.0	00.3	05.9	25.4	67.6	00.1
Inflation	00.0	02.2	00.4	00.3	21.5	57.7	13.8	04.1
Nominal interest rate	00.2	22.2	01.3	01.4	26.5	07.2	01.9	39.3
Real interest rate	00.0	01.4	00.0	00.1	02.2	05.2	22.1	69.0
4-quarter forecast error decom.								
Financial premium	100.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0
Consumption	02.3	74.3	00.6	00.2	07.9	01.6	01.6	11.6
Investments	12.5	04.1	00.3	68.4	05.7	00.9	01.8	06.3
Output	02.0	21.0	11.8	33.7	10.1	02.7	02.4	16.4
Labour	01.1	13.7	07.7	13.7	50.4	01.4	02.0	10.0
Employment	04.1	13.5	07.5	17.6	26.1	02.4	07.4	21.4
Real wages	00.0	01.7	00.0	02.6	09.2	18.4	67.0	01.1
Inflation	00.0	03.8	00.5	00.3	22.2	39.8	23.3	10.1
Nominal interest rate	00.6	22.4	01.4	02.3	30.3	09.3	08.1	25.8
Real interest rate	00.5	08.7	00.4	01.7	12.8	12.5	06.4	57.0
10-quarter forecast error decomp.								
Financial premium	100.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0
Consumption	04.7	61.3	01.1	02.6	11.5	01.6	04.3	12.8
Investments	25.6	06.4	00.6	48.2	07.9	00.7	04.0	06.7
Output	06.2	13.4	07.4	31.7	15.4	02.2	06.7	17.0
Labour	03.4	12.2	06.9	13.1	43.2	01.6	06.1	13.5
Employment	10.3	08.5	05.9	13.9	14.6	02.3	17.4	27.0
Real wages	00.3	01.5	00.0	11.5	13.6	15.7	54.1	03.2
Inflation	00.2	03.6	00.5	00.7	21.0	38.3	23.8	12.1
Nominal interest rate	01.8	23.1	01.6	02.5	29.1	07.4	12.7	21.8
Real interest rate	02.5	10.9	00.7	02.9	17.0	10.8	06.8	48.5
Unconditional variance decomp.								
Financial premium	100.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0
Consumption	10.7	42.9	01.0	21.3	09.2	01.1	04.3	09.5
Investments	34.5	06.2	00.7	41.0	07.1	00.6	04.2	05.8
Output	13.2	12.3	06.2	30.6	13.9	01.9	07.3	14.6
Labour	05.7	11.4	06.3	16.6	39.6	01.5	06.4	12.6
Employment	16.4	06.7	04.2	25.8	10.9	01.7	15.0	19.4
Real wages	12.7	02.5	00.2	29.3	10.8	09.2	31.9	03.3
Inflation	04.3	03.4	00.4	02.3	19.7	35.8	22.5	11.5
Nominal interest rate	05.3	21.2	01.5	06.2	26.6	06.8	12.3	20.1
Real interest rate	04.5	10.7	00.7	03.6	16.5	10.4	06.8	46.8

Table 9 Variance decomposition of *version C* of the SWFA model

	ε_f	ε_c	ε_g	ε_i	ε_a	ε_p	ε_w	ε_r
1-quarter forecast error decom.								
Financial premium	88.4	00.2	00.1	10.5	00.6	00.0	00.0	00.2
Consumption	02.4	75.4	00.6	01.0	07.5	00.9	00.8	11.4
Investments	18.5	00.7	00.0	74.8	01.1	00.9	00.0	04.0
Output	01.5	28.6	27.2	27.2	02.8	01.7	00.2	10.8
Labour	00.5	10.1	09.8	09.6	65.5	00.4	00.2	03.8
Employment	04.8	16.1	09.2	09.5	44.5	01.7	02.1	12.0
Real wages	00.0	00.6	00.0	00.3	07.1	25.5	66.3	00.1
Inflation	00.1	02.9	00.3	00.2	21.6	57.2	13.1	04.5
Nominal interest rate	00.6	26.5	01.5	00.9	25.9	06.3	01.7	36.6
Real interest rate	00.0	00.9	00.0	00.0	04.2	07.0	21.5	66.4
4-quarter forecast error decom.								
Financial premium	81.2	01.4	00.2	14.5	01.8	00.0	00.3	00.7
Consumption	04.7	56.9	01.1	02.4	15.7	01.8	03.2	14.3
Investments	46.2	01.3	00.0	42.1	02.5	01.7	00.3	05.9
Output	06.6	23.1	11.9	23.5	11.8	03.5	01.9	17.8
Labour	03.9	16.1	08.3	11.0	45.1	02.0	01.9	11.8
Employment	11.7	17.5	08.1	07.0	28.4	02.9	05.2	19.1
Real wages	00.0	01.8	00.0	02.2	13.1	18.9	62.7	01.3
Inflation	00.2	05.2	00.4	00.2	23.6	38.8	21.2	10.5
Nominal interest rate	02.2	28.6	01.4	00.8	29.6	07.8	07.0	22.5
Real interest rate	01.4	11.1	00.5	00.7	11.7	12.2	06.4	56.0
10-quarter forecast error decomp.								
Financial premium	71.7	04.4	00.4	17.1	04.1	00.1	01.0	01.3
Consumption	07.1	41.0	01.6	07.2	21.6	01.5	06.4	13.6
Investments	71.6	00.8	00.0	20.4	01.8	01.1	00.4	03.9
Output	18.7	15.8	08.4	17.1	15.9	03.0	04.3	16.7
Labour	10.2	14.3	07.6	10.2	37.7	02.1	04.4	13.5
Employment	24.5	13.0	07.1	05.2	19.1	02.4	09.7	19.0
Real wages	01.0	01.8	00.0	07.0	20.9	16.2	50.0	03.2
Inflation	00.4	05.1	00.4	00.4	22.4	37.5	21.3	12.5
Nominal interest rate	05.5	30.7	01.5	01.0	28.6	05.3	10.5	16.9
Real interest rate	06.1	16.9	00.9	00.7	18.3	09.1	06.8	41.3
Unconditional variance decomp.								
Financial premium	67.0	06.5	00.5	17.4	05.4	00.1	01.6	01.5
Consumption	17.1	33.3	01.4	11.4	18.5	01.2	06.0	11.1
Investments	69.1	02.5	00.2	20.8	02.7	00.8	00.7	03.2
Output	30.3	13.6	07.2	14.6	13.7	02.5	03.9	14.0
Labour	14.1	13.1	06.9	12.8	34.6	02.0	04.2	12.3
Employment	32.8	09.8	05.4	13.3	15.6	01.9	07.6	13.6
Real wages	28.1	01.7	00.1	08.4	15.9	10.8	32.5	02.6
Inflation	07.1	05.6	00.5	00.5	21.2	34.2	19.4	11.4
Nominal interest rate	10.7	28.8	01.5	02.9	26.0	04.8	10.1	15.2
Real interest rate	08.9	17.4	00.9	02.3	17.7	08.3	06.9	37.6

Figure 1a Financial shock

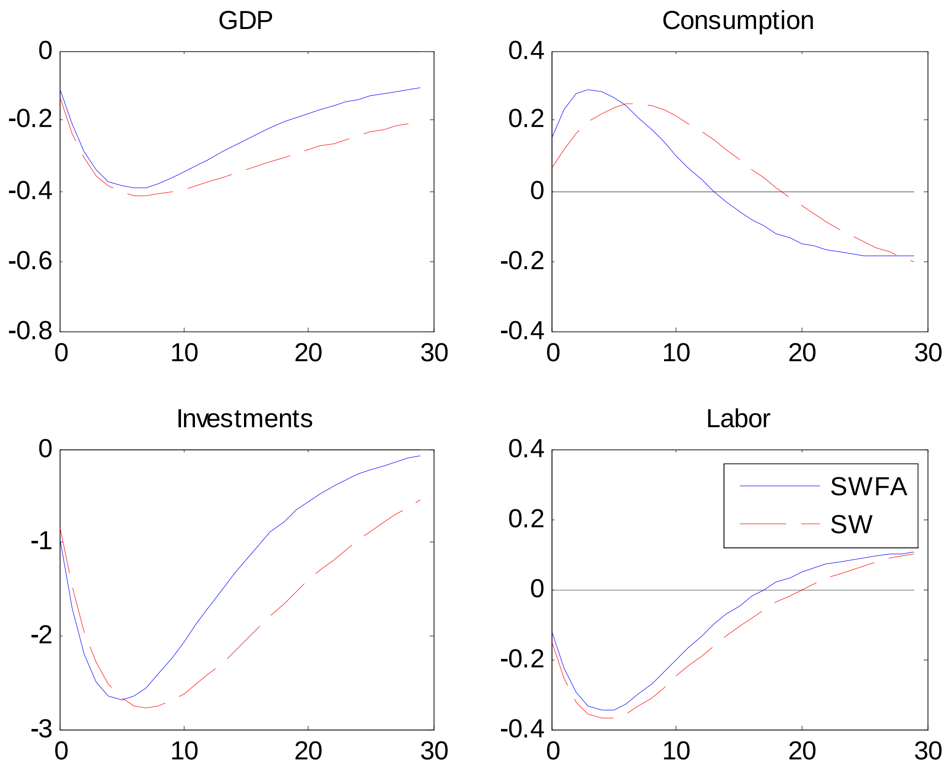

Figure 1b Financial shock

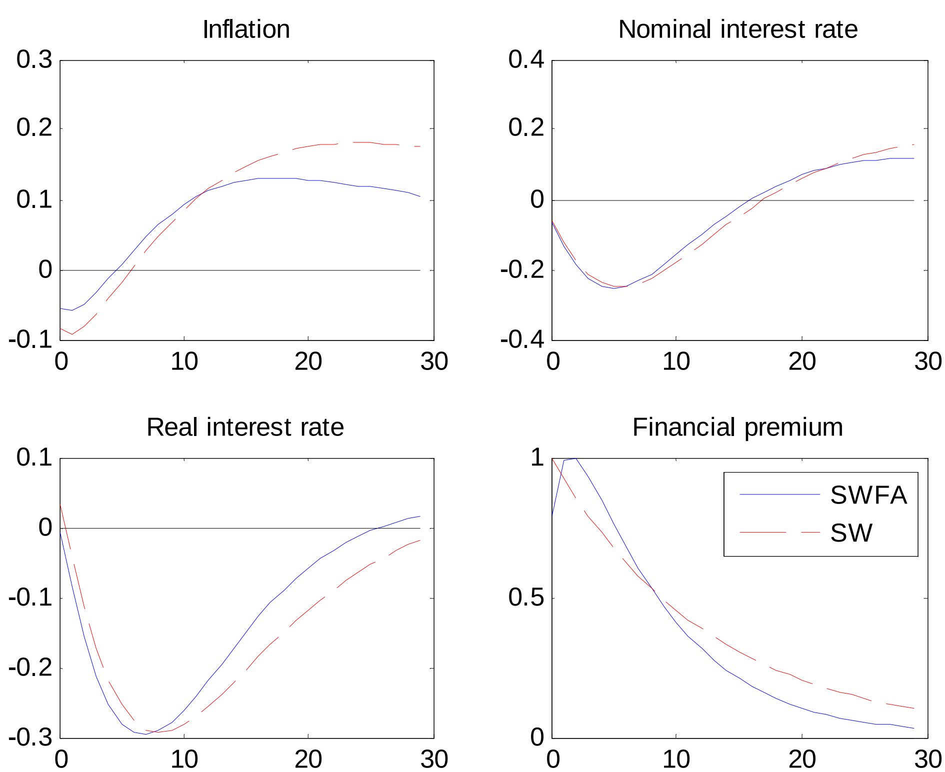

Figure 1c Financial shock

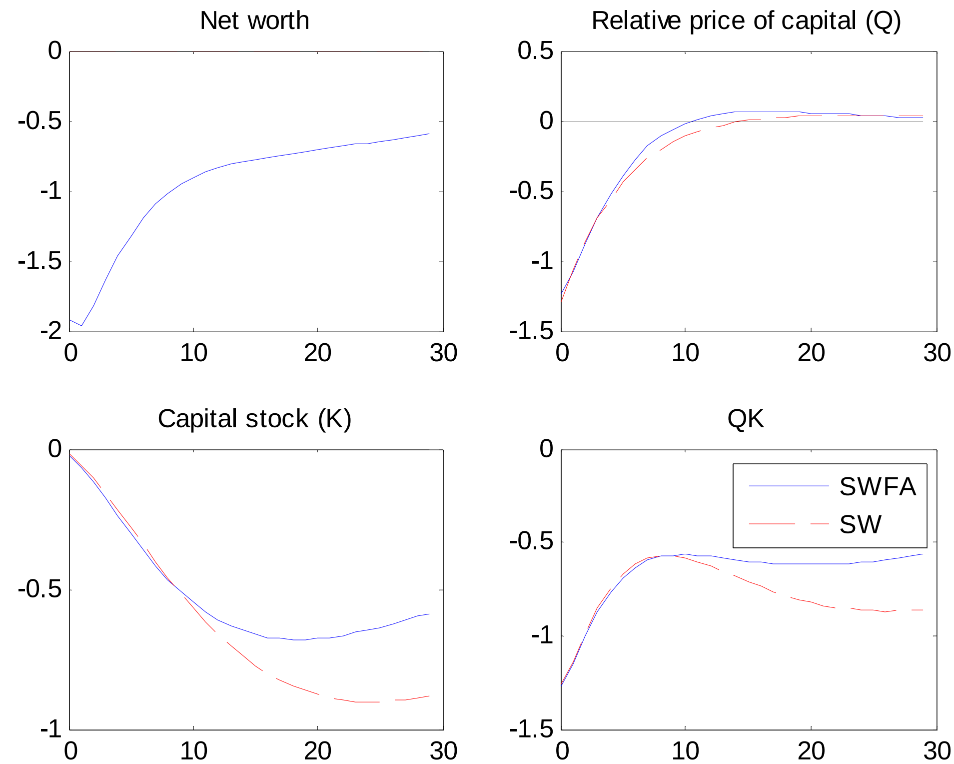

Figure 2a Preference shock

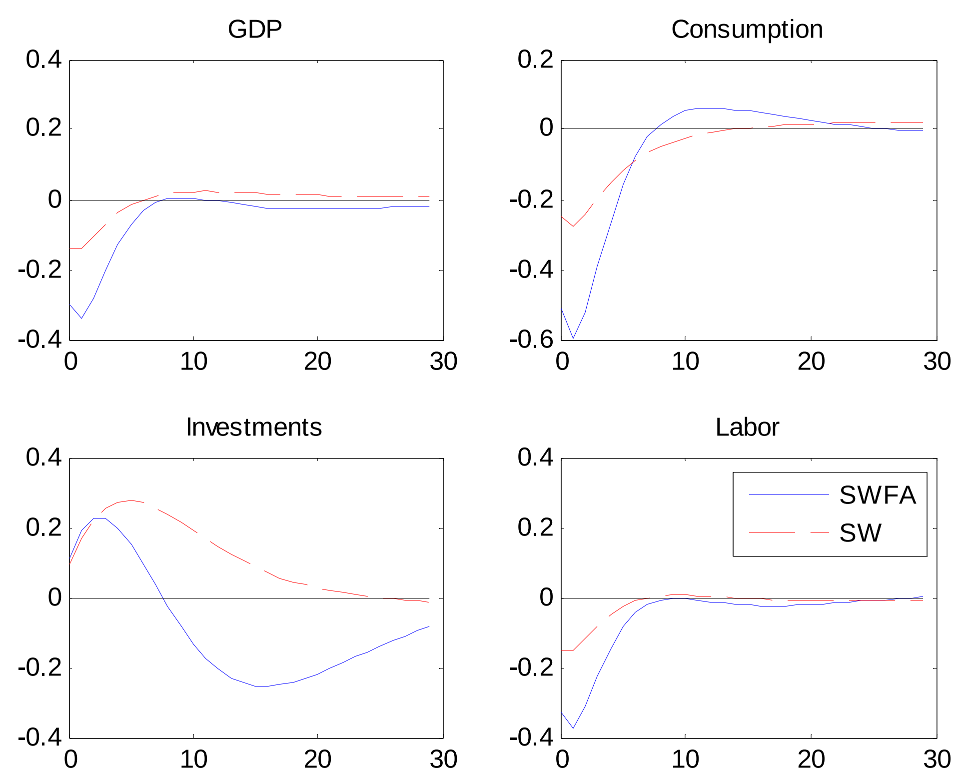

Figure 2b Preference shock

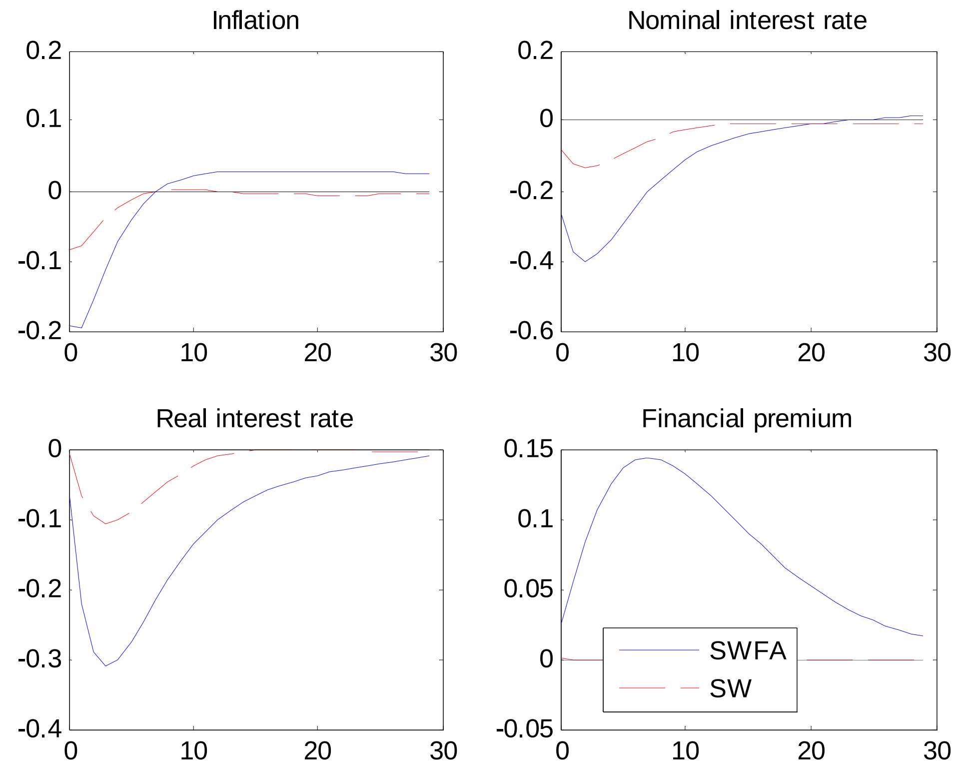

Figure 2c Preference shock

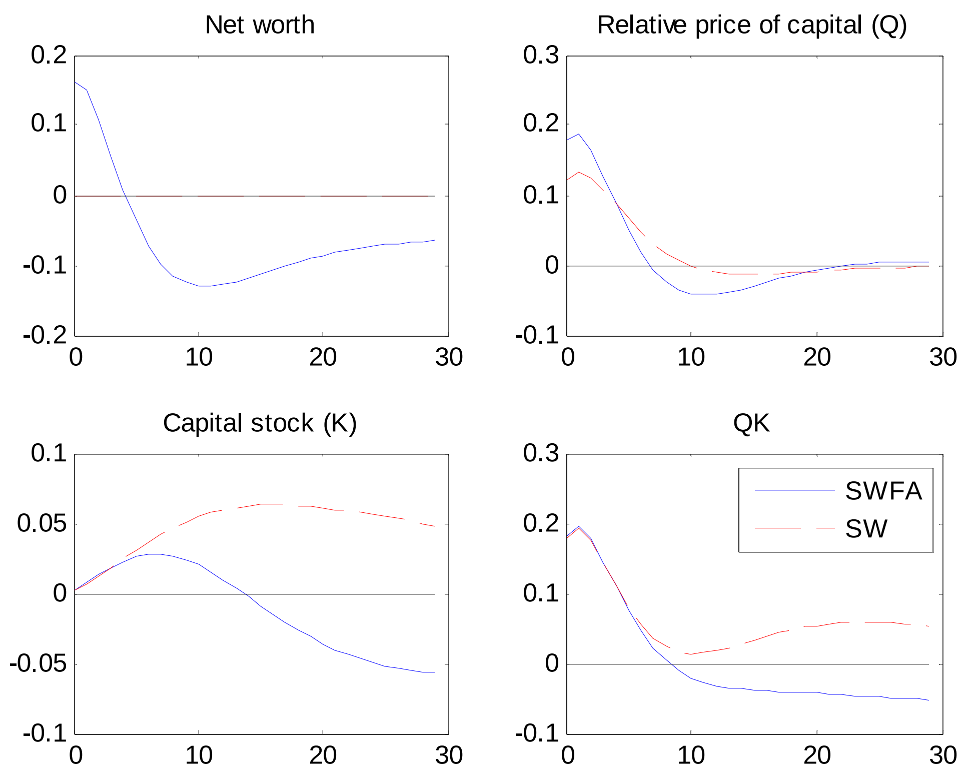

Figure 3a Government-spending shock

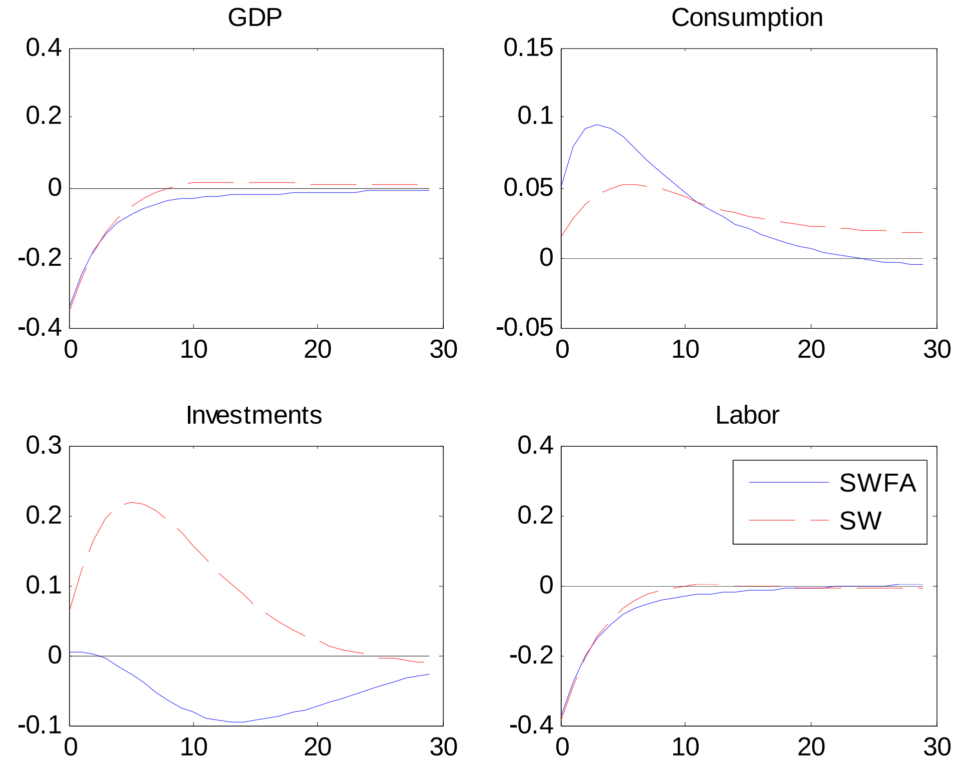

Figure 3b Government-spending shock

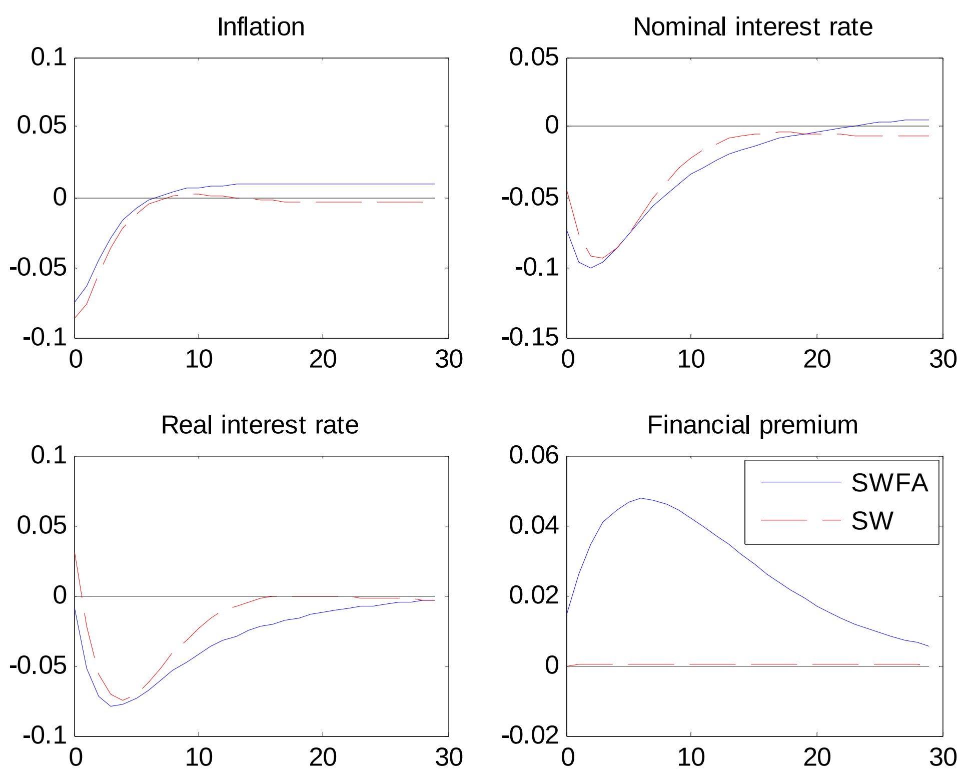

Figure 3c Government-spending shock

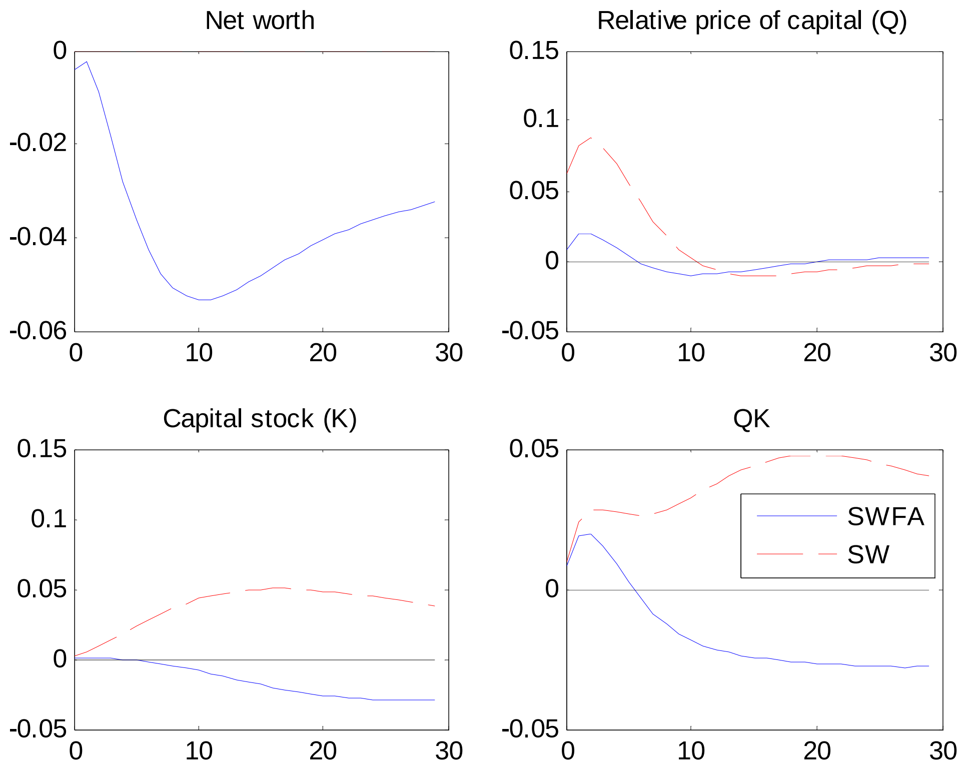

Figure 4a Investments shock

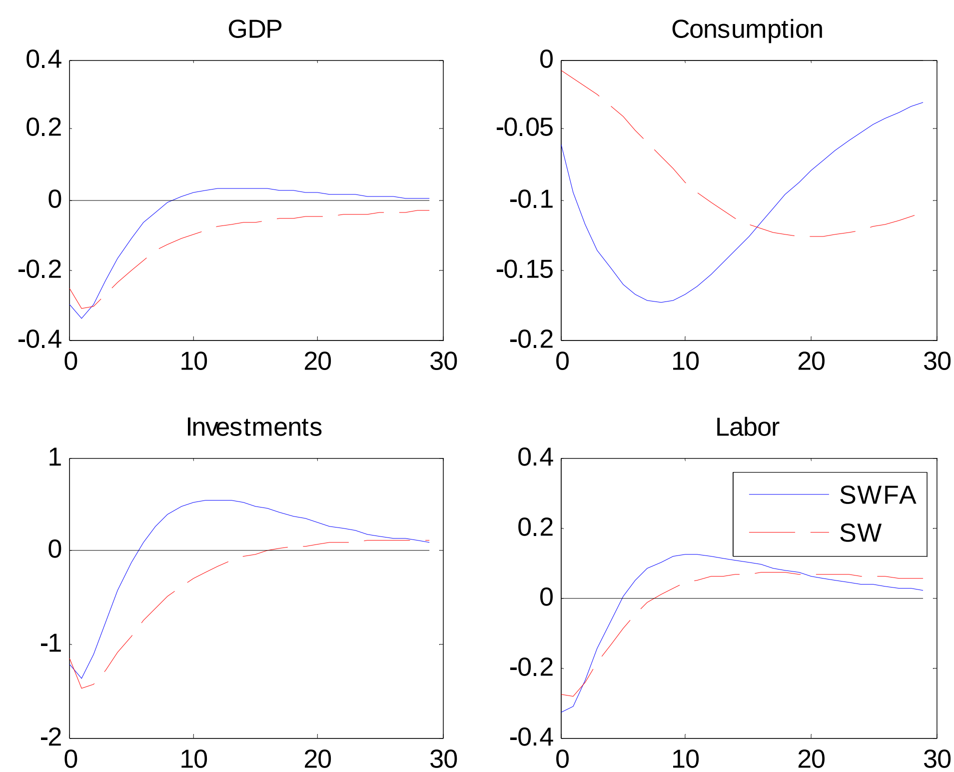

Figure 4b Investments shock

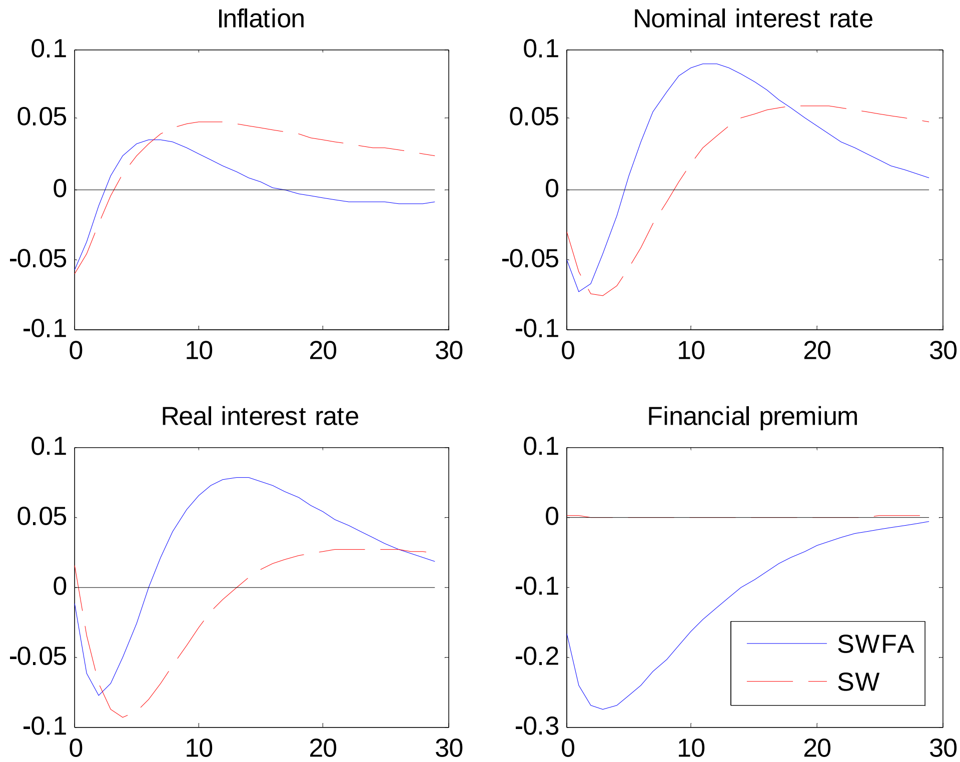

Figure 4c Investments shock

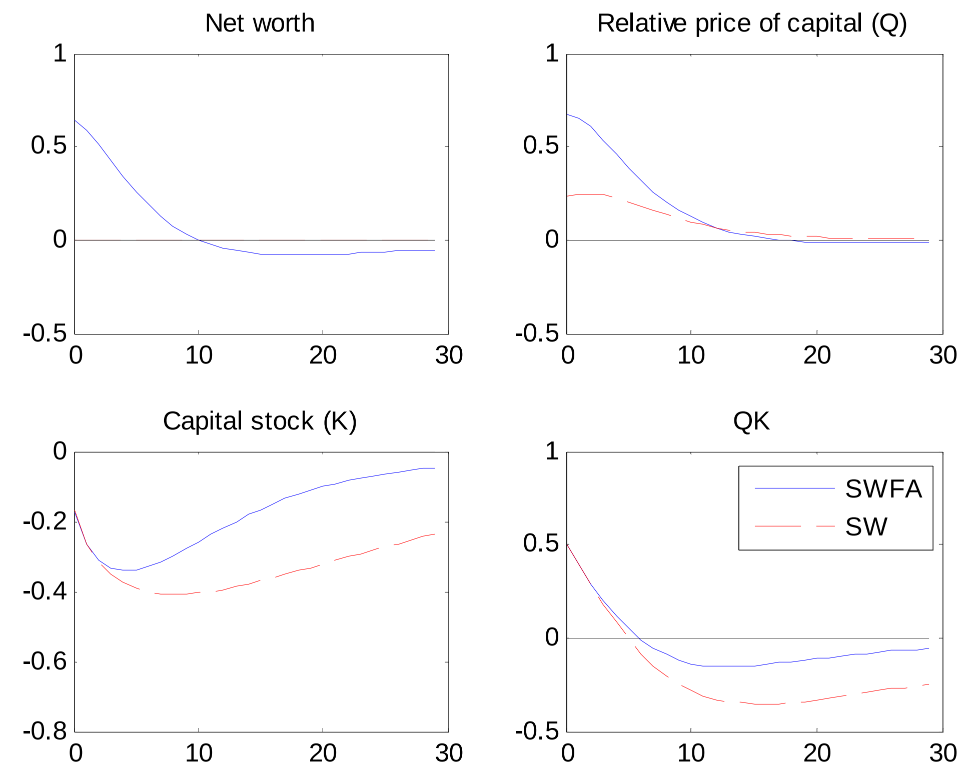

Figure 5a Monetary-policy shock

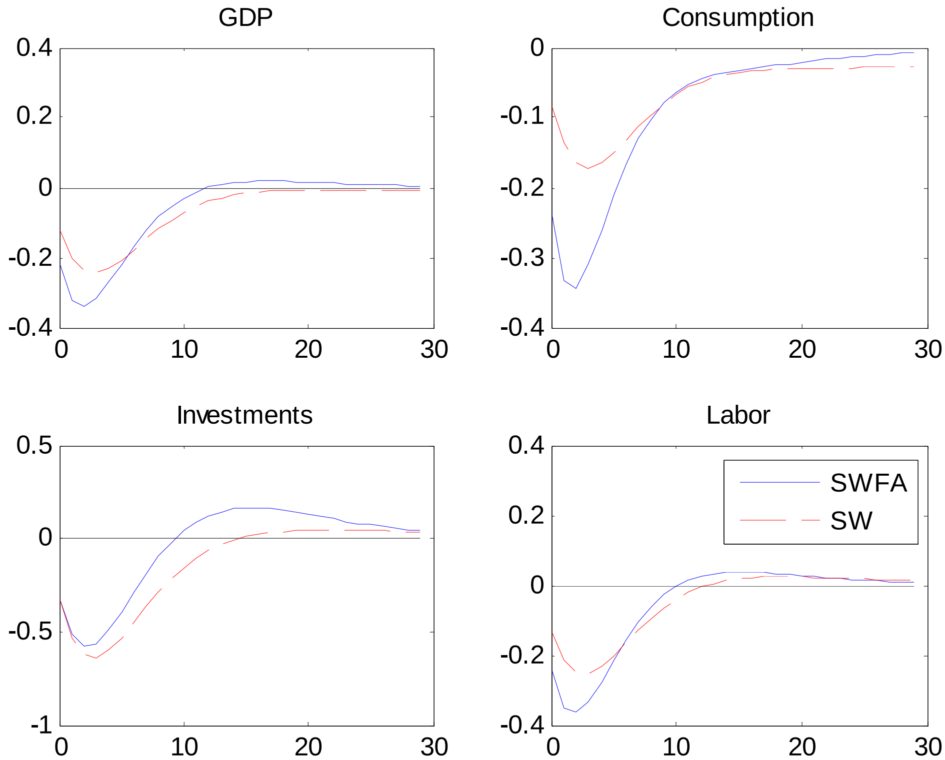

Figure 5b Monetary-policy shock

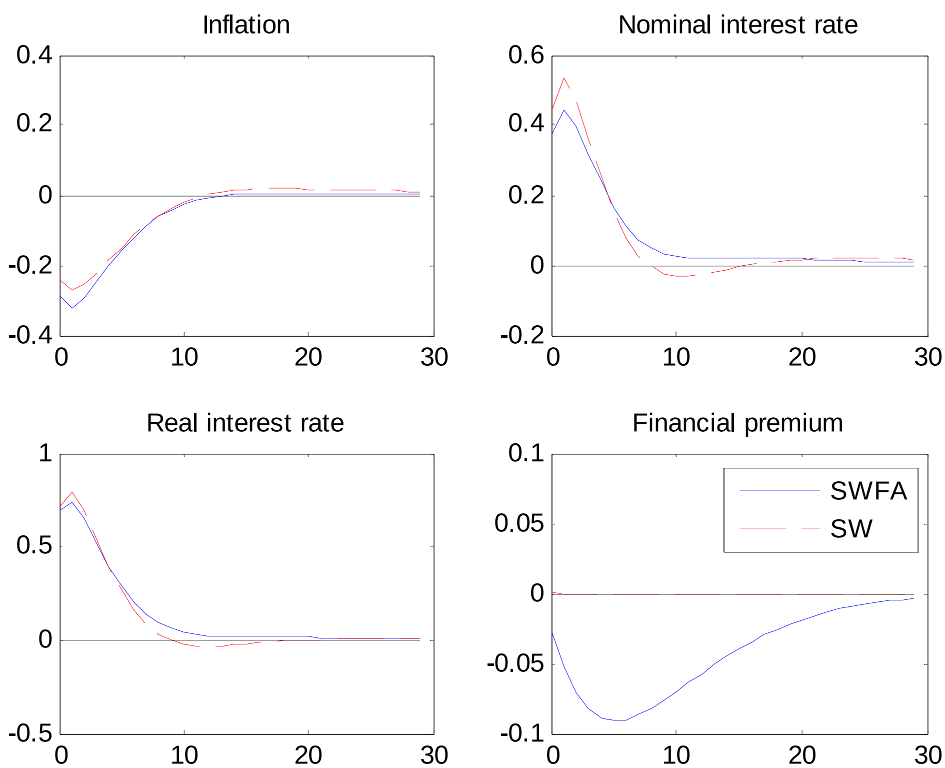

Figure 5c Monetary-policy shock

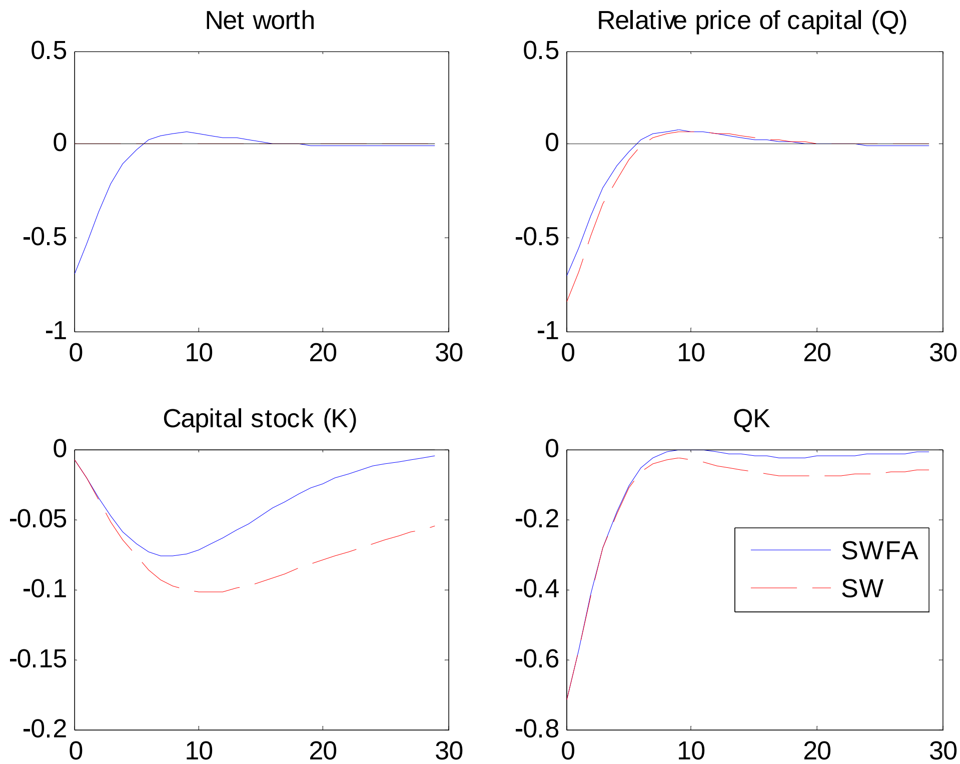

Figure 6a Productivity shock

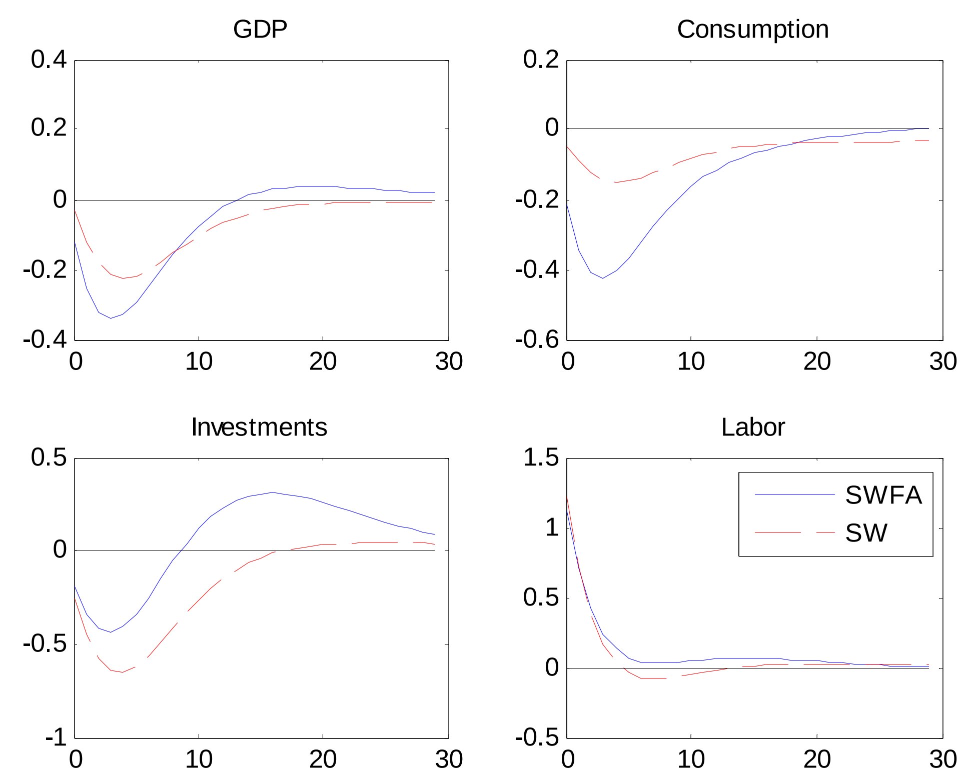

Figure 6b Productivity shock

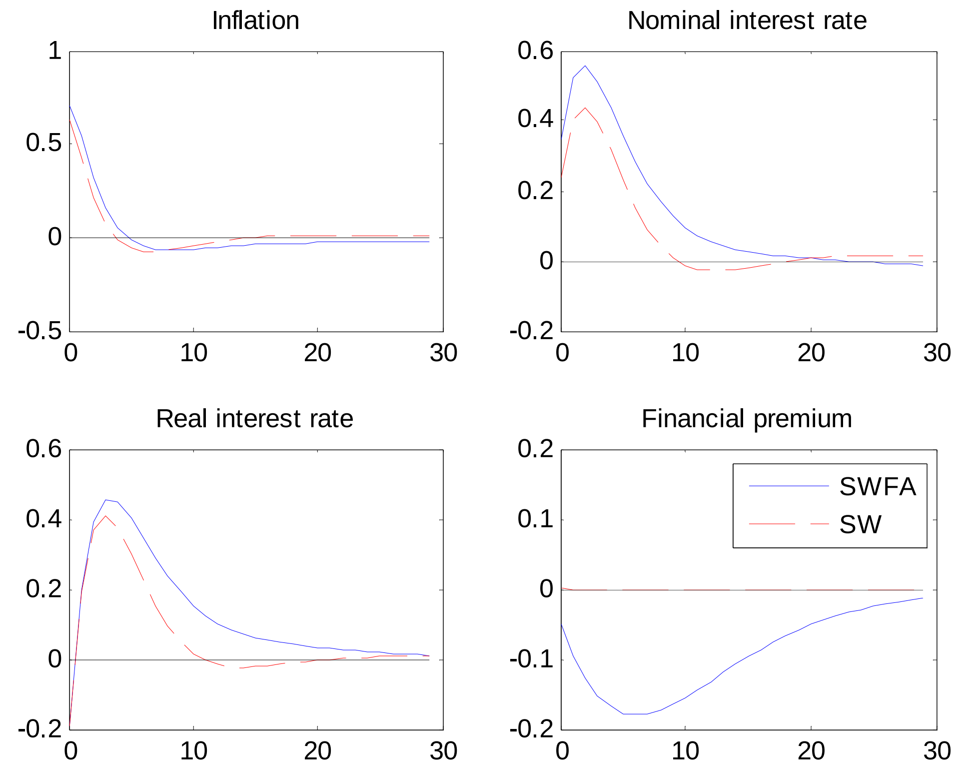

Figure 6c Productivity shock

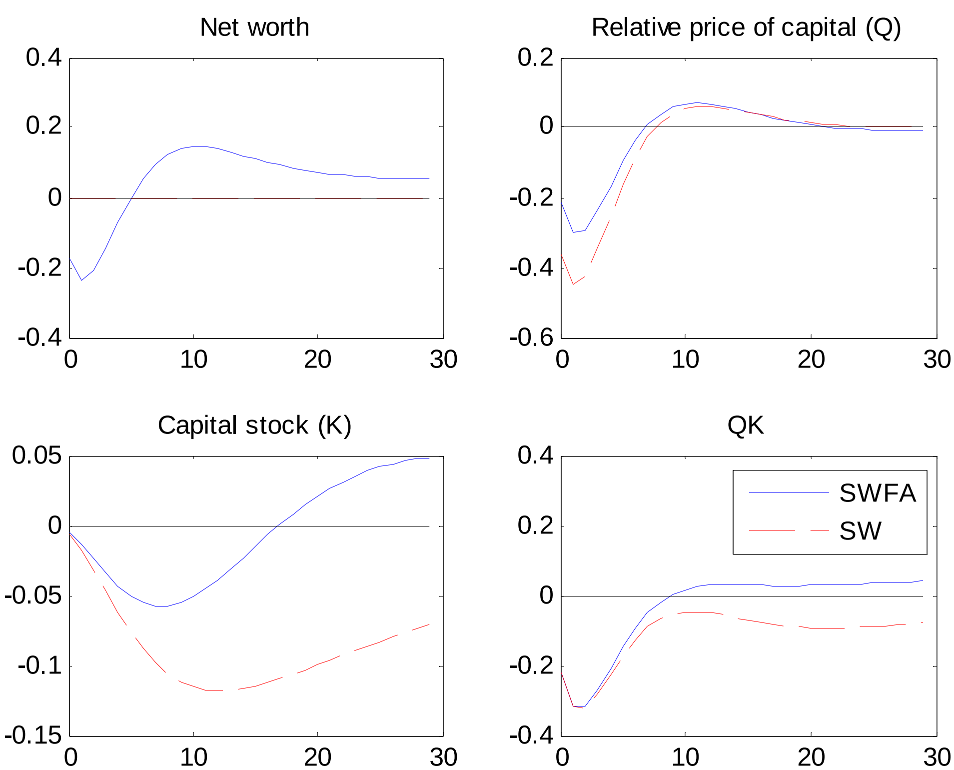

Figure 7a Price-markup shock

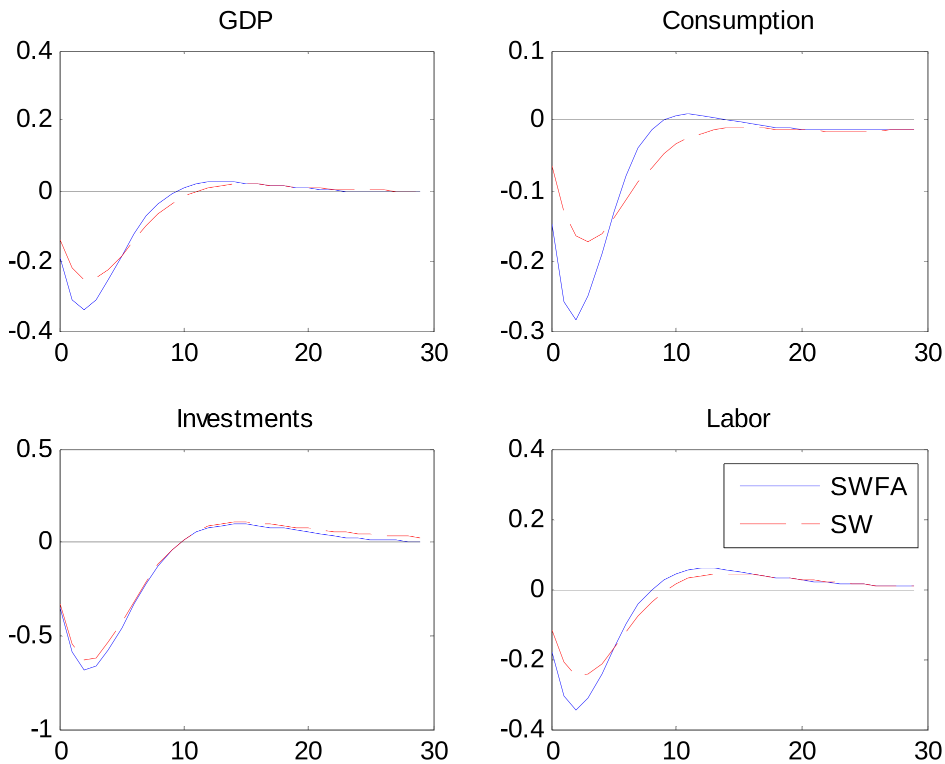

Figure 7b Price-markup shock

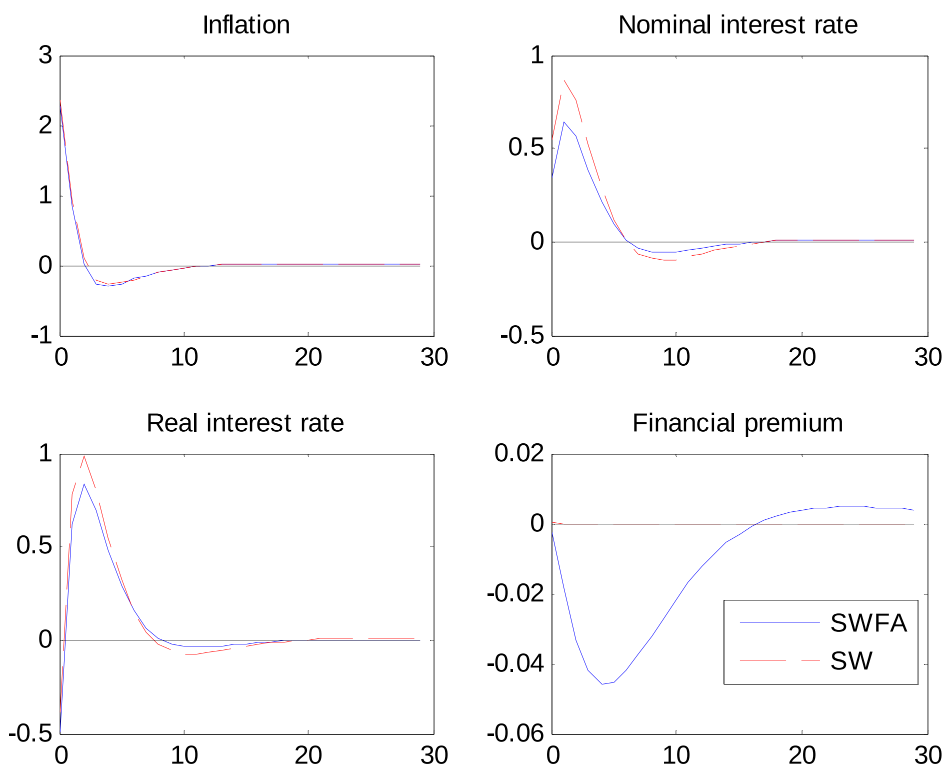

Figure 7c Price-markup shock

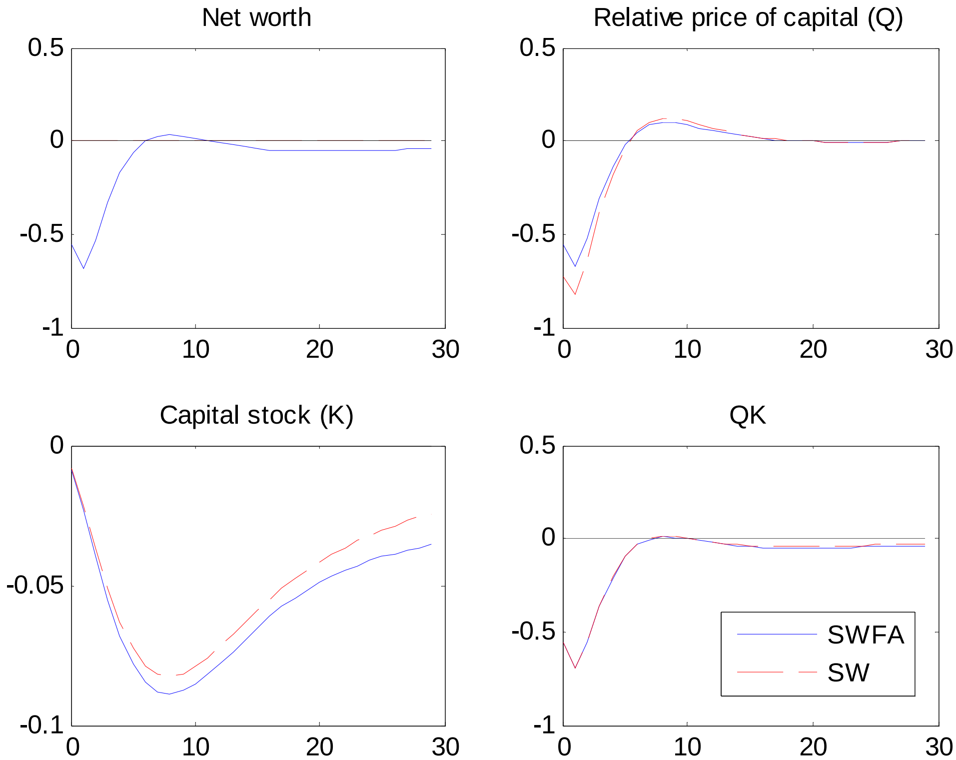

Figure 8a Wage-markup shock

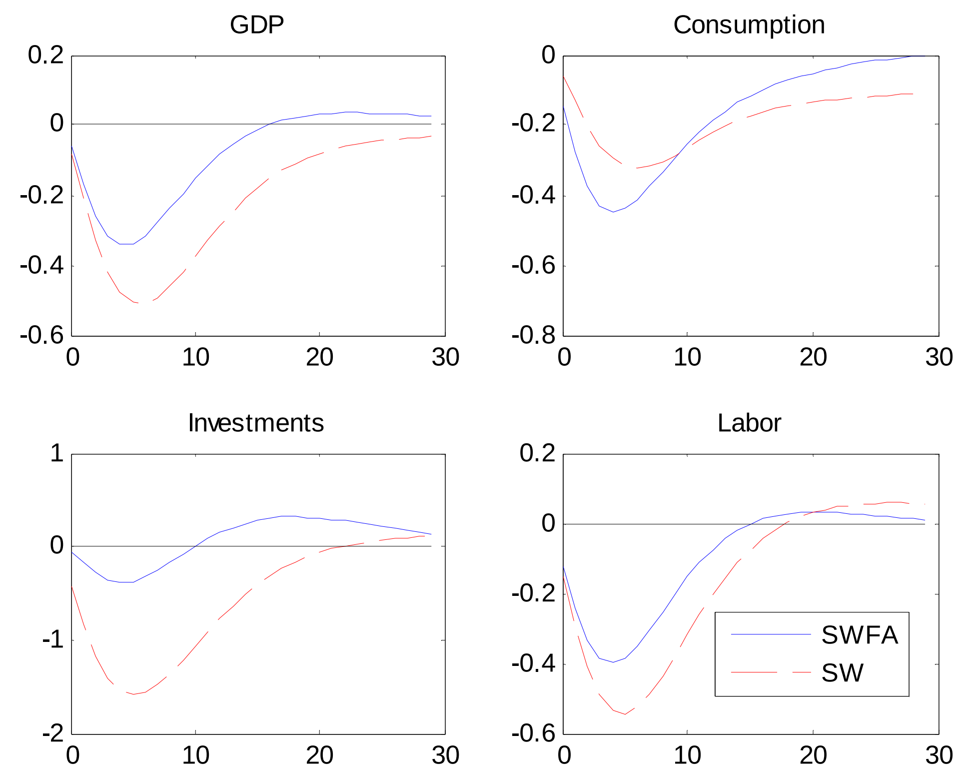

Figure 8b Wage-markup shock

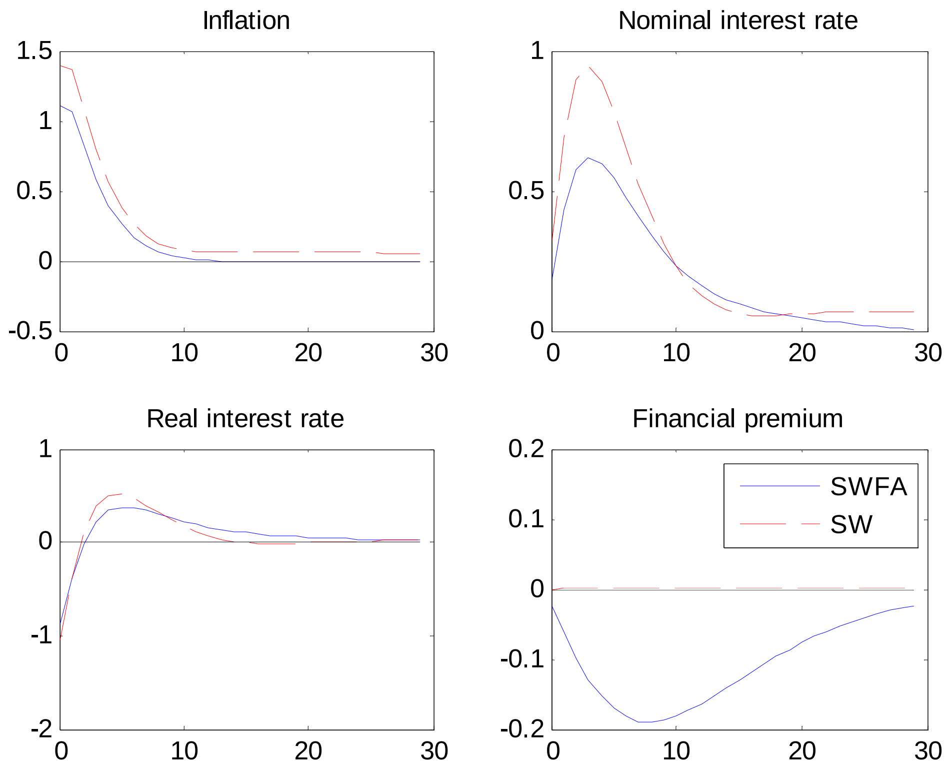

Figure 8c Wage-markup shock