Wage Rigidity and Labor Market Dynamics in Europe and the United States*

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Abstract

This paper takes the perspective of the search and matching model of the labor market to analyse real wage rigidity. In particular, we estimate a real general equilibrium business cycle model with search frictions in the labor market, and determine the relative degrees of wage rigidity for newly hired and incumbent workers. We conduct this exercise for both U.S. and Euro Area data and find that, while incumbent workers' wages are less rigid in the U.S., the rigidity of wages for newly hired workers is negligible in both areas. Our results are thus in line with micro studies that cannot find the rigidity of wages needed to explain high unemployment volatility in models with search and matching frictions. However, the estimation points at driving forces other than the shocks to productivity commonly used to assess the performance of these models.

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1 Introduction

The key element of the search and matching model are rents to long-term employment relationships generated by matching frictions. How these rents are shared determines workers' labor income and firms's profit incomes, and at the same time the incentives for firms to post vacancies to form new matches. When wages in the model are continuously renegotiated by Nash bargaining, they strongly respond to improving labor market conditions and thus mute firms' incentives to post vacancies in a cyclical upswing. Simulated with productivity movements that are as volatile as in the data, the model cannot replicate the volatility of unemployment.[1]

This 'unemployment-volatility puzzle' (Pissarides, 2009) can in principle be resolved by assuming real wage rigidity for both new and incumbent workers, as argued by Shimer (2005) and Hall (2005). Such an assumption has been employed in a number of subsequent papers, arguing that new hires' wages need to be consistent with firms' overall wage structure, due, for example, to internal equity constraints.[2] It is the behavior of the wages of new hires that is crucial for how firms' incentives to post vacancies adjust, and thus for the response of unemployment to shocks.

Recently, Haefke, Sonntag, and van Rhens (2008) and Pissarides (2009) have shown that micro data are not consistent with the assumption of rigid wages for newly hired workers. In particular, they find that those wages move about as much as predicted by the matching model without wage rigidity. In other words, the wage rigidity needed to generate high unemployment volatility in the search and matching model can in fact not be found in the data, so the answer must lie in some other mechanism.

In this paper, we use U.S. and Euro Area data to estimate a canonical business cycle model with search and matching in the labor market, and letting the data decide on the relative degrees of wage rigidity for newly hired and incumbent workers

[1] See Costain and Reiter (2008), Hall (2005), Shimer (2005), and Pissarides (2009) for expositions of the key issues. Andolfatto (1996) is an early paper identifying the difficulty of the flexible wage version of the search and matching model in explaining unemployment volatility.

[2] Examples are Gertler, Sala, and Trigari (2008) and Krause and Lubik (2007). An important reference containing interview evidence on wage rigidity is Bewley (1999).

in each region. Furthermore, the estimation determines how the variation in output, unemployment and wages is explained by three stochastic disturbances, namely shocks to productivity, price markups, and the matching process. We find that, while incumbent workers' wages are less rigid in the U.S. than in the Euro Area, the rigidity of wages for newly hired workers is negligible in both economic areas. Our results are thus in line with micro studies that cannot find the rigidity of wages needed to explain high unemployment volatility in models with search and matching frictions. However, the estimation points at driving forces other than the shocks to productivity commonly used to quantitatively assess the performance of these models.

We model wage rigidity along the lines of Bodart, Pierrard, and Sneessens (2006) and Gertler and Trigari (2007), who assume Calvo-style (1983) wage setting. That is, only a fraction of wages can be renegotiated every period, and the possibility to adjust arrives stochastically. So each match has a given, time-invariant probability that the wage can be adjusted to changing conditions. Firms and workers take this into account during bargaining. Furthermore, wages in newly formed matches are not freely negotiated, but, with a certain probability, need to correspond to the wage of incumbent workers. While this typically refers to the wages of the existing workforce of firms, we take the economy-wide average wages as the reference point. These assumptions result in an aggregate wage equation that determines current period wages as a weighted average of past wages and newly negotiated wages.

For the U.S. we find that, across specifications, the degree of incumbent workers' wage rigidity implies a probability of not adjusting wages of about 0.46, while the rigidity of new hires' wages is a mere 0.01. For the Euro Area, the degree of wage stickiness is higher, at about 0.60, and that for new hires at about 0.07.Essentially, this means that in both regions, new wages are freely negotiated, but then adjust with the probabilities of incumbent workers.[3] Thus, the estimation suggests an average duration of wage stickiness of about 1.8 quarters (5.4 months) in the U.S. and 2.5 quarters (7.5 months) in the Euro area, which is comparatively low compared to other evidence.

[3] Only in one specification, where we fix worker bargaining power, do we find a higher, albeit imprecise degree of wage rigidity for new hires.

In terms of variance decomposition, the model assigns most variation in the unemployment rate to a shock in the matching process, rather than to productivity shocks, even though the latter shock is the main driver of output. This result means that the search and matching model does not generate endogenously the hiring flows needed to explain employment movements in response to the incentives to produce. Instead, it resorts to a shock which independently moves employment. Therefore, in the standard model, either the matching function or the hiring cost function or both are possibly incorrectly specified. Given independent evidence on the matching function as a Cobb-Douglas function like the one assumed here, it appears that rather the hiring costs are not linear. Furthermore, this may mean that the intertemporal Euler equation for the job creation decision is incorrect. Basically, the labor market and the product market are disjoint in the model.

Vacancy creation is the driver of employment (and unemployment) fluctuations in the search-and-matching model. Including data on vacancies in the estimation is obviously an important way to assess the empirical performance of the model. Unfortunately, such data are not available for the Euro Area as a whole, so we have to restrict attention in this case to the U.S. Nevertheless, when we include vacancy data in the estimation, the model's performance improves significantly. An important reason is that the additional shock we include, which makes workers' outside option stochastic, turns out to be central for job creation. It takes over the role of the matching function shock in generating unemployment fluctuations. However, the estimated degrees of wage rigidity remain largely unaffected. Since this shock can be interpreted as an exogenous change in labor supply, our findings is in line with other results in the literature that estimates DSGE models.

In the next section, we present the baseline search and matching model, along with the assumptions on wage setting. We show under which conditions the rigidity of newly hired wages affects the dynamics of labor market adjustment. Section 3 discusses the (Bayesian) estimation procedure, presents the results, and conducts robustness checks. Finally, section 4 concludes. A appendices describe the data, ans show model equations and derivations not detailed in the text.

2 A simple business cycle model with labor market search

We begin the description of our model economy by characterizing the labor market frictions that make the search of workers and firms for trading partners costly. Then we show how workers and firms determine the value of search and the value of a match. Since both trading partners derive a surplus from being in a productive match, a bilateral monopoly problem must be resolved by bargaining. A typical assumption is that the joint surplus of the parties is divided according to the Nash bargaining solution, which chooses that wage as the solution to the problem, which maximizes the joint surplus.

We modify the wage bargaining assumption by using the specification of Bodart et al. (2006), who introduce Calvo-type rigidities in the wage setting process. The key assumption that we adopt is that the probability of being able to set the wage may differ between new hires and existing jobs. For existing jobs, the wage is assumed to be fixed until the firm-worker pair receives the signal that they can renegotiate. For new matches, wages are either freely negotiated with a certain probability or otherwise set according to the previous period's economy-wide wage. The parties are forward looking, so they take account of the fact that wages may stay fixed for some time.

We assume that the economy consists of two sectors: an intermediate good sector that produces a homogenous good using labor as the only input, and a final goods sector in which monopolistically competitive firms use the intermediate good to produce differentiated products sold to households. The product differentiation gives market power to each monopolist, leading to a markup of prices over marginal costs. While the main thrust of our analysis concerns a real economy, we include price setting to allow for exogenous markup variations, which can be interpreted as a type of demand shock.[4]

[4] In a model with price stickiness, that is, if the model is of the new Keynesian type, this markup variation would endogenously arise.

2.1 Final good

There is a continuum of households who maximize the present value of consumption

$$\max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t C_t$$

where C_t is an aggregate of differentiated product varieties given by

$$C_t = \left[\int_0^1 c_t(i)^{1-\frac{1}{\sigma_t}} \, di \right]^{\frac{\sigma_t}{\sigma_t - 1}}, \quad \sigma > 1,$$

Demand for an individual variety is isoelastic, with the (time-varying and exogenous) demand elasticity given by σ_t. Since prices are flexible, the monopolistically competing variety producers simply set their price as a markup over marginal cost. The marginal cost is the price of the homogenous intermediate good, x_t.

We take the consumption aggregate as the numeraire. Since all final variety producers set the same price, those are also equal unity. Let $\mu_t = \sigma_t / (\sigma_t - 1)$ be the gross mark-up, then the intermediate price must satisfy

$$x_t = \frac{1}{\mu_t}.$$

Allowing for a time-varying mark-up (and hence demand elasticity) is a simple way of introducing demand-type fluctuations into the model. The shocks to the relative price x_t are potentially important for explaining labor market fluctuations, as argued in Krause, Lopez-Salido, and Lubik (2008), Rotemberg (2008), and Lubik (2009). We let the data to decide on the importance of this shock.

For simplicity, we assume that utility is linear in the final consumption aggregate. Households simply maximize the discounted present value of income, and they are risk neutral with respect to work and unemployment. Each household is assumed to consist of a large number of members that can be either employed or unemployed, depending on the hiring and firing behavior of firms. Each member is assumed to be willing to work. The period income is given by

$$y_t = w_t n_t + b_t (1 - n_t),$$

where w is the wage income, b is the monetized benefit flow while unemployment, and n_t is the fraction of household members that are employed. With linear utility, both households and firms discount the future with the constant discount factor β. In principle, the outside option b_t can be time-dependent - we return to this issue later.

2.2 The labor market

The homogenous intermediate good is produced by firms that hire workers from a frictional labor market. The central elements of the search and matching model of the labor market (Mortensen and Pissarides, 1994) are the costly search of workers and firms that arises from search frictions, and a matching function that represents these frictions. The inputs to the function are workers' and firms' search activity, and the output is the number of matches – newly hired workers – formed in a period. This is analogous to a production function with unemployed workers and vacant jobs as inputs.

We assume that the matching function is a Cobb-Douglas function that gives the flow of new matches as:

$$m_t = \bar{m} e^{\varepsilon_t^m} v_t^{\vartheta} u_t^{1-\vartheta},$$

where u_t is the number of searching workers, n_t is employment, v_t is the measure of vacancies, and ε_t^m is a shock to matching productivity. The labor force is normalized to one.

Existing employment relationships separate at an exogenously given job destruction rate ρ. Thus, aggregate employment evolves according to

$$n_t = (1-\rho)\, n_{t-1} + m_t.$$

Given the constant returns assumption for the matching function, the match flow m_t can be expressed as a function of labor market tightness, $\theta_t = v_t/u_t$, namely

$$\begin{aligned} q_t &= \frac{m_t}{v_t} = \bar{m} e^{\varepsilon_t^m} \theta_t^{\vartheta-1} \\ s_t &= \frac{m_t}{u_t} = \bar{m} e^{\varepsilon_t^m} \theta_t^{\vartheta}. \end{aligned}$$

Then, $m_t = v_t q(\theta_t)$.

Note that we follow a number of authors who make the simplifying assumption that hiring is instantaneous. In other words, increased search activity by firms, a higher v_t, leads to a contemporaneous increase in hirings. This mainly simplifies the analysis of the model in certain respects, but does not have any substantive effects on the results.[5]

More precisely, our timing assumption is as follows. First, at the beginning of the period separations take place and vacancies are posted. Next, matches are formed and hiring decisions are made. Third, wage bargaining takes place whenever the new and existing matches receive a favorable signal to do so. Finally, production takes place. Workers are not able to search in the same period as separations occur. This means that the number of searching workers is given by

$$u_t = 1 - n_{t-1}.$$

2.2.1 Workers

Employment status is achieved when drawing a match with a firm. As explained above, the wage may or may not be negotiated at a particular time period. Let w_t^* indicate the wage that is being set flexibly by the firm and worker (we detail the bargaining assumption below). The value of employment to a worker in a job with newly set wages is given by

$$W_t\left(w_t^*\right) = w_t^* + \beta E_t\left\{(1-\rho)\left[\gamma_w W_{t+1}\left(w_t^*\right) + \left(1-\gamma_w\right) W_{t+1}\left(w_{t+1}^*\right)\right] + \rho U_{t+1}\right\}, \quad (1)$$

where γ_w is the probability of not being able to negotiate the wage in existing jobs and U_t is the value of unemployment. Notice that workers rationally foresee that the wage may stay fixed for some time.

In those new jobs, where the wage cannot be bargained for, it is simply set to the previous period's average wage level. Using w_t to denote the economy-wide average

[5] See Gertler and Trigari (2008), Rotemberg (2007), and Blanchard and Gali (2009). Most importantly, the assumption of instantaneous hiring allows an analysis of the New Keynesian sticky price model without introducing an additional margin that allows the instantaneous adjustment of output to nominal demand shocks.

wage, the value of a job for a worker is written as

$$W_t\left(w_{t-1}\right)=w_{t-1}+\beta E_t\left\{(1-\rho)\left[\gamma_w W_{t+1}\left(w_{t-1}\right)+\left(1-\gamma_w\right) W_{t+1}\left(w_{t+1}^*\right)\right]+\rho U_{t+1}\right\}. \tag{2}$$

Finally, the value of unemployment is given by

$$U_t=b_t+\beta E_t\left\{s_{t+1}\left[\vartheta_w W_{t+1}\left(w_t\right)+\left(1-\vartheta_w\right) W_{t+1}\left(w_{t+1}^*\right)\right]+\left(1-s_{t+1}\right) U_{t+1}\right\}, \tag{3}$$

where b indicates any source of utility that is only available when unemployed, i.e., principally unemployment benefits and leisure. Here ϑ_w is the Calvo probability of not being able to negotiate the wage in a new match, and $s_t=m_t/u_t$ is the job finding rate.

2.2.2 Firms

The value of a filled job to firms can be written similarly to the value for workers. Again, we distinguish new matches with negotiated wages and new matches with pre-set wages. The two value functions are given by

$$J_t\left(w_t^*\right)=x_t e^{z_t}-w_t^*+\beta(1-\rho) E_t\left[\gamma_w J_{t+1}\left(w_t^*\right)+\left(1-\gamma_w\right) J_{t+1}\left(w_{t+1}^*\right)\right] \tag{4}$$

and

$$J_t\left(w_{t-1}\right)=x_t e^{z_t}-w_{t-1}+\beta(1-\rho) E_t\left[\gamma_w J_{t+1}\left(w_{t-1}\right)+\left(1-\gamma_w\right) J_{t+1}\left(w_{t+1}^*\right)\right], \tag{5}$$

respectively; z_t is an aggregate productivity process with mean zero, so that output per match fluctuates around one.

Firms post vacancies if it is profitable to do so. Assuming free entry, the value of a new vacancy

$$V_t=-\kappa+q_t\left[\vartheta_w J_t\left(w_{t-1}\right)+\left(1-\vartheta_w\right) J_t\left(w_t^*\right)-V_t\right]$$

is driven to zero, which implies:

$$\frac{\kappa}{q_t}=\vartheta_w J_t\left(w_{t-1}\right)+\left(1-\vartheta_w\right) J_t\left(w_t^*\right). \tag{6}$$

Notice that here too we account for the possibility that wages in filled vacancies may not be negotiated, so with a probability ϑ_w the new match will have the pre-set wage w_{t-1}. Recall that hiring is instantaneous, so that current values appear on the right-hand side of (6).

2.2.3 Wage bargaining

We need to specify how wages are set when a match has the possibility to negotiate them. We resolve the bilateral monopoly situation between negotiating worker and firm by assuming the Nash bargaining solution. This is standard in the literature, and our only modification is that the value functions take into account the subsequent partial fixity of renegotiated wages. Formally, the bargaining outcome is a solution to the following problem:

$$\max_{w_t^*} \left[W_t\left(w_t^*\right) - U_t\right]^{\eta} J\left(w_t^*\right)^{1-\eta},$$

where the parameter η captures the bargaining power of workers. The first-order condition to the problem results in a sharing rule:

$$\eta J_t\left(w_t^*\right) = (1-\eta)\left[W_t\left(w_t^*\right) - U_t\left(w_t^*\right)\right]. \tag{7}$$

which specifies the wage as the one that shares the value of the match between worker and firm according to the weights η. Typically, the value of a job is pinned down by the entry condition when $\vartheta_w = 0$. Then the sharing rule can be seen to give workers a surplus $W - U$ proportional to $\eta/(1-\eta)$ of the value of a new job. With $\vartheta_w > 0$, vacancies have a chance to produce a value that is determined by average wages, and thus the close link between new jobs' values and worker surplus breaks down.

2.3 Equilibrium

The typical way to solve search-and-matching models with flexible wages is to derive a condition for the wage rate and another for job creation. We follow the same route, but since the derivation is tedious, we relegate it to the Appendix. Here we only list the resulting equations, and discuss their interpretation.

First, we note that average wages are a weighted sum of pre-set wages, and newly bargained wages in existing jobs and new matches. Formally, the following equation describes the evolution of average wages:

$$w_t = \frac{m_t}{n_t}\left[\vartheta_w w_{t-1} + (1-\vartheta_w)\, w_t^*\right] + \frac{(1-\rho)\, n_{t-1}}{n_t}\left[\gamma_w w_{t-1} + (1-\gamma_w)\, w_t^*\right]. \tag{8}$$

Next, using the Nash rule (7) and the definitions of the value functions, the newly renegotiated wage w_t^* for new hires and incumbent workers can be shown to be

$$w_t^* = \omega_t + E_t \sum_{j=1}^{\infty} \left[\beta\gamma_w (1-\rho)\right]^j \left[w_{t+1}^* - w_t^* - \frac{s_{t+1}\vartheta_w}{(1-\rho)\,\gamma_w}\left(w_{t+1}^* - w_t\right)\right], \tag{9}$$

where ω_t is the "shadow Nash wage" that is identical to the wage that would be obtained without wage rigidity:

$$\omega_t = \eta\left(x_t e^{z_t} + \beta\kappa E_t \theta_{t+1}\right) + (1-\eta)\, b_t \tag{10}$$

This is the wage that would be the outcome if there were no wage rigidities, i.e., $\gamma_w = \vartheta_w = 0$. As in the standard search and matching model, it is a weighted average of the value of the match to the firm and the outside option of the worker. Because of the timing assumption on employment flows, we do not allow workers who would separated from a job during bargaining to be rehired in the same period. This is why next period's θ_t enters the Nash wage equation.[6]

In our rigid wage setting the average wage reflects not only current values of these variables, but also future expected labor market conditions. Wage rigidity tilts newly negotiated wages away from the "shadow Nash wage", depending on whether newly set wages are expected to rise or fall. There are two forces at work. In the presence of wage rigidity for existing workers, an expected rise in the new wage would lead to higher wages today. Breaking up negotiations would force a firm to pay a higher wage anyways. If wage rigidity for new hires is present, $\vartheta_w > 0$, then the expected difference between future new wages and current wages would act towards reducing wages today. The reason is that the higher the rigidity of newly set wages, the lower

[6] Of course, such separations during bargaining never happen in equilibrium, but the threat to do so is part of the bargaining process.

is the benefit of breaking up negotiations to obtain a higher wage in the future. Conversely, firms benefit. So while the first term in square brackets in (9) reflects the force of higher expected new wages on current wages, the second term reflects the countervailing force of sticky wages for new hires. Interestingly, the new wage differs from the shadow Nash wage by a infinite discounted sum of the wage differences from the current period to the next. No further information is needed.

It is instructive to see how the newly negotiated wage, w_t^*, is set in the case of $\vartheta_w = 0$. The newly set wage then can be solved for explicitly, and is given by

$$w_t^* = \left[1 - \beta\gamma_w(1-\rho)\right] \sum_{j=0}^{\infty} \left[\beta\gamma_w(1-\rho)\right]^j E_t\omega_{t+j}. \tag{11}$$

Thus w^* is simply a weighted average of current and future Nash wages, or more concretely current and future labor market conditions. In particular, the new wage responds less than proportionately to a productivity shock, as long as that shock is not permanent. We explore the dynamics of w_t^*, w_t and ω_t in more detail below.

Substituting the newly hired wage from (8), and log-linearizing yields the following approximate equation for real wage inflation:

$$\hat{\pi}_t^w = \frac{\beta\left[(1-\rho)\gamma_w - \bar{s}\vartheta_w\right]}{\rho\vartheta_w + (1-\rho)\gamma_w} E_t\hat{\pi}_{t+1}^w + \frac{\left[1 - \rho\vartheta_w - (1-\rho)\gamma_w\right]\left[1 - \beta\gamma_w(1-\rho)\right]}{\rho\vartheta_w + (1-\rho)\gamma_w}\left(\hat{\omega}_t - \hat{w}_t\right), \tag{12}$$

where hats denote log deviations from steady state values. Thus the evolution of the average real wage can be interpreted as given by a Phillips-curve, where the driving force is the deviation of the Nash wage as given by (10) from the the actual wage. When $\vartheta_w = 0$, the wage inflation is given by the rather familiar wage Phillips curve

$$\hat{\pi}_t^w = \beta E_t\hat{\pi}_{t+1}^w + \frac{(1-\widetilde{\gamma}_w)(1-\beta\widetilde{\gamma}_w)}{\widetilde{\gamma}_w}\left(\hat{\omega}_t - \hat{w}_t\right),$$

where $\widetilde{\gamma}_w = (1-\rho)\gamma_w$. One can see that new wage rigidity reduces both the contemporaneous effect of the deviation of the Nash wage and the actual wage (i.e., the slope of the Phillips curve), and the effect of future wage inflation. The latter of course reflects future deviations between Nash and actual wage.

The job creation condition follows from (6) and the equations for the value functions and the wage rates:

$$
\begin{aligned}
\frac{\kappa_t}{q(\theta_t)} &= x_t e^{z_t} - \omega_t + (1-\rho)\, E_t \beta_{t+1} \frac{\kappa_{t+1}}{q(\theta_{t+1})} \\
&\quad - \frac{\vartheta_w}{\gamma_w} \left(1 - \frac{s_{t+1}}{(1-\rho)}\right) \left(w^*_{t+1} - w_t\right) \sum_{j=1} \left[\beta\,(1-\rho)\,\gamma_w\right]^j \\
&\quad + \vartheta_w (w^*_t - w_{t-1}) E_t \sum_{j=0} \left[\beta\,(1-\rho)\,\gamma_w\right]^j
\end{aligned}
$$

Similar to the wage equation, incentives for job creation are muted by the behavior of the Nash wage ω. Thus the first line above looks exactly as in the standard search and matching model. In the presence of real wage rigidities, other terms enter, but only when there is real wage rigidity for new hires, $\vartheta_w > 0$. The rigidity of existing workers wages does not matter per se, unless, of course, it is zero.

The log-linear version of the equation is given as

$$
\begin{aligned}
\hat{q}_t &= \beta\,(1-\rho)\, E_t \hat{q}_{t+1} - \frac{\bar{q}}{\kappa} \left[\bar{x}\bar{z}\,(\hat{x}_t + z_t) - \bar{w}\hat{\omega}_t\right] \\
&\quad + \frac{\bar{q}}{\kappa} \frac{\vartheta_w \bar{w}}{\left[1 - \beta\gamma_w\,(1-\rho)\right]\left[1 - \vartheta_w \rho - \gamma_w\,(1-\rho)\right]} E_t \left[\beta\,(1-\bar{s})\,\hat{\pi}^w_{t+1} - \hat{\pi}^w_t\right].
\end{aligned} \tag{13}
$$

Notice also here that when the wages of new hires are flexible ($\vartheta_w = 0$), the last term disappears and the job creation condition is independent of the wage rigidity of existing jobs, but governed by the notional Nash wage ω_t specified above. This is the point made, for example, by Pissarides (2009). In fact when wages of new hires are flexible, the job creation condition is identical to the case of perfectly flexible wages, given by the Nash wage in (10).

Rigidity of new hires' wages, on the other hand, does influence job creation. When $\vartheta_w > 0$, firms vacancy posting desision is influenced by the expected evolution of wages. Since w_t is a state variable in this case, postponing hiring one period means that the pre-set wage (in case of not being able to negotiate) changes from w_{t-1} to w_t. A higher expected wage inflation next period would, for example, encourage hiring today, in order to lock in the currently lower wage. The third term thus captures the value of waiting depending on the future evolution of the wage rate.

To close the model, we need to specify market clearing for intermediate goods.

Since utility is linear and labor supply is inelastic[7], the total value of output is given residually as

$$y_t = e^{\varepsilon_t^z} n_t,$$

which is a function of aggregate productivity and the number of employed workers. Thus there are three structural shocks: a productivity shock, ε_t^z, a shock to the matching function, ε_t^m, and a shock to the price markup, ε_t^μ, which derives from the randomness of the demand elasticity of differentiated goods, σ_t. In this model without price rigidity, this amounts to a shock to the wage equation.

3 Estimating wage rigidity

Now we turn to our main exercise, which is the estimation of the wage rigidity parameters. In this section, we first describe the data sets used and our estimation and calibration strategy. We do not estimate all parameters but choose them based on other sources of information. Then we present our results, and finally conduct robustness exercises.

3.1 Data

For the estimation, we use the Area Wide Model dataset for the Euro Area (see Fagan, Henry and Mestre, 2001, update 2006), and publicly available data from the BLS and from the St. Louis FED for the US. We include the US because it has a longer and more reliable time series, so we can use it to cross-check the European results. Secondly, we are interested in comparing the US and the EA in terms of wage rigidity, and wage rigidity for new hires in particular. We detail data construction in the Appendix.

We use detrended data, employing the Hodrick-Prescott filter with a smoothing parameter of 1,600. The sample period is1982:Q1-2005Q4 for the Euro Area and 1982Q1-2008Q3 for the US. The end of the samples is determined by availability. We pick 1982 as the starting point because (i) EA data is not available before 1970

[7] This is in the sense that all potential workers are either searching for a job or are employed, and hours are supplied inelastically.

(and naturally it is synthetic data for long afterwards), and (ii) we do not want the turbulence of the 1970s to have an influence on the estimation results. Nevertheless, we experimented with different samples, and our main conclusions remained robust.

Since our focus is on the labor market, we use only three variables: the real wage, the unemployment rate, and real GDP. While the search-and-matching model with a constant separation rate operates through job creation, we do not use data on vacancies, because such data is not available for the Euro Area as a whole. Thus, we are forced to drop it also for the US in the main section. Later on we perform a robustness check with vacancy data for the US.

3.2 Calibration

We calibrate most of the parameters of our model, since our focus is on the rigidity of wages. This also allows us to circumvent the identification problems inherent in the search and matching model.[8] We set the discount factor to $\beta = 0.992$, which implies a steady-state annualized real interest rate of 3.27. The elasticity of the matching function is chosen to be $\vartheta = 0.5$, which is a typical value in the literature and is also in the range values deemed admissible by Pissarides and Petrongolo (2001). We assume a prior bargaining power on $\eta = 0.5$, which gives workers and firms equal bargaining strength. We leave this parameter to be estimated. The markup is set at 10 percent, that is, $\mu = 1.1$ in steady state.

We calibrate the unemployment rates to equal the steady state rates observed over the sample period, which is $\bar{u} = 0.092$ for the Euro Area, and $\bar{u} = 0.056$ for the U.S. The job finding rates are taken from Hobijn and Sahin (2007), who calculate them at a monthly frequency for OECD countries. We compute the Euro Area value as a population weighted mean; we convert the monthly rates to a quarterly frequency in all cases. This way we get a job finding rate of $\bar{s} = 0.158$ for the EA and $\bar{s} = 0.92$ for the US. From the steady state employment flow condition we can calculate the separation rate, which gives $\rho = 0.016$ for the EA and $\rho = 0.054$ for the US. These values confirm the common belief that US job turnover is much faster, and both

[8] See Lubik (2009) for a thorough discussion.

European jobs and unemployment last much longer.

The replacement rate is set at the value $b = 0.4$ advocated by Shimer (2005), who calibrated it for the US. We choose a slightly higher value of $b = 0.6$ for Europe, but we allow in a robustness check this value to be estimated for both regions, giving it a prior mean of $b = 0.5$. Given values for $\bar{s}$, b, and ρ, the steady state wage and job creation conditions can be solved for $\bar{w}$ and $\kappa\bar{\theta}$, which are the only remaining values needed in the log-linear equations. Thus, fortunately, we do not need vacancy data or data on job filling probabilities which are difficult to calculate.

Now we turn to the parameters we estimate: Table 1 contains the priors for the estimation. We specify relatively loose priors, especially in the case of the Calvo parameters. Thus we rely on the data to identify these key parameters as much as possible.

Parameters	**Prior mean**	**Prior s.d.**	**Distribution**
ϑ_w	0.5	0.2	Beta
γ_w	0.5	0.2	Beta
η	0.5	0.2	Beta
ρ_z	0.5	0.15	Beta
ρ_{mf}	0.5	0.15	Beta
ρ_μ	0.5	0.15	Beta
$s.d.\varepsilon^z$	0.05	0.05	Inv. Gamma
$s.d.\varepsilon^{mf}$	0.05	0.05	Inv. Gamma
$s.d.\varepsilon^\mu$	0.05	0.05	Inv. Gamma

Table 1: Bayesian priors

We have three shocks in the model: a productivity shock, a markup shock in the price equation, and a shock to the efficiency of match creation. The stochastic processes all have a prior persistence with a mean of 0.5, and a prior standard deviation of 0.05. These priors in turn have the standard deviations given in the table. For the persistence parameters we assume a Beta distributions, while the standard deviations follow an Inverse Gamma Distribution.

3.3 Results

This subsection presents in detail the results of our baseline estimation scenario, while the next subsection discusses variations in the parameters chosen to be estimated, as well as the effects of including measures of vacancies in the estimation.

3.3.1 The baseline scenario

In the baseline, we estimate the two degrees of wage rigidity for both regions, along with the bargaining parameter η. Table 2 presents our baseline estimation results. The following findings stand out. First, both types of wage rigidity (for new and incumbent workers) are lower in the U.S. than in the Euro Area, and significantly different from each other, and in the case of the Euro Area, also from the prior. However, new hires' wage rigidity is virtually zero in both regions. On the other hand, the bargaining power η is significantly larger than the prior of 0.5. The persistences of shocks differ across regions, and is lower in the Euro Area. Only technology shocks appear similarly persistent. Interestingly, also the standard deviations of technology shocks and matching function shocks are similar across regions. Markup shocks are less volatile but more persistent in the U.S.

The estimation results are in many aspects close to preconceptions about U.S. and Euro Area labor markets. The Euro Area appears more rigid in terms of both bargaining power and wage stickiness. European workers earn a larger share of match surplus, thus tending to suppress the responsiveness hiring, and thus employment, to shocks. At the same time, the higher rigidity of wages should work towards amplifying labor market dynamics.

Overall, it appears that newly hired workers almost always freely negotiate their wages with employers, irrespective of prevailing wages in the economy. Or course, it should be noted that the wages of new workers are expected to remain fixed for the same duration as those of incumbent workers. But there appears to be no additional constraint on new workers wages, arising from existing wages of the workforce. We want to emphasize that this is not a literal statement about actual individual wage setting procedures for workers in the data. Rather, our findings imply that real wage

Parameter	**Prior mean**	**Posterior mean**	**90% Conf. interval**
		Euro Area	
γ_w	0.50	0.6036	$0.5465 - 0.668$
ϑ_w	0.50	0.0786	$0.0045 - 0.1546$
η	0.50	0.8206	$0.7029 - 0.9173$
ρ_z	0.5	0.5299	$0.3581 - 0.7287$
ρ_{mf}	0.5	0.5560	$0.4320 - 0.684$
ρ_μ	0.5	0.3263	$0.1989 - 0.4658$
$s.d.\varepsilon^z$	0.05	0.0080	$0.0078 - 0.0083$
$s.d.\varepsilon^{mf}$	0.05	0.0419	$0.0366 - 0.0469$
$s.d.\varepsilon^\mu$	0.05	0.0224	$0.0140 - 0.0295$
		US	
γ_w	0.50	0.4638	$0.3933 - 0.5430$
ϑ_w	0.50	0.0251	$0.0014 - 0.0513$
η	0.50	0.7637	$0.6433 - 0.8944$
ρ_z	0.5	0.5741	$0.4346 - 0.7249$
ρ_{mf}	0.5	0.8801	$0.8158 - 0.9424$
ρ_μ	0.5	0.6162	$0.5088 - 0.7286$
$s.d.\varepsilon^z$	0.05	0.0081	$0.0078 - 0.0083$
$s.d.\varepsilon^{mf}$	0.05	0.0395	$0.0351 - 0.0440$
$s.d.\varepsilon^\mu$	0.05	0.0144	$0.0113 - 0.0173$

Table 2: Estimation results

rigidity in a simple, (essentially) representative agent search and matching model does not help understanding unemployment and wage dynamics at the aggregate level.

3.3.2 Simulating the model

We simulate the model using the modes of the estimated posterior distributions of the parameters. The first purpose of the simulation is to see if the model is able to reproduce basic stylized facts. For this we feed the model with a shock process that has the properties estimated from the data, but whose realizations differ from that implied by the empirical data. We report standard deviations, cross-correlations, and autocorrelations. The model does reasonably well along these dimensions, at least for the observed variables. Secondly, we report the theoretical variance decomposition based on the estimated parameters and shock processes. We show that the model performs less well here, as unemployment volatility comes basically from the matching function shock.

Tables 3, 4 and 5 report standard deviations, cross-correlations and autocorrelations for the three observables: GDP, unemployment, and the real wage. The estimated model does a reasonably good job in replicating these data moments, which - given the simplicity of the model and the shock stochastic processes - is not tautological.

	Euro Area		U.S.	
	Data	Model	Data	Model
Unemployment	4.40	4.30	8.63	8.73
Real Wage	0.60	0.70	0.92	1.18
GDP	0.68	1.04	0.96	1.16

Table 3: Standard deviations

The standard deviations of y, u and w are quite well reproduced, although the model somewhat overpredicts the volatility of the real wage, and fairly significantly (for the Euro Area) the volatility of GDP. While unemployment volatility is almost perfectly matched, we show below that it is because of the lack of internal propagation in the model.

Cross-correlations are qualitatively and quantitatively largely reasonable: the

	Euro Area					
	Data			Model		
	GDP	Unemp.	Real wage	GDP	Unemp.	Real wage
GDP	1			1		
Unemp.	−0.74	1		−0.46	1	
Real wage	0.4	−0.47	1	0.48	−0.22	1
	United States					
	Data			Model		
	GDP	Unemp.	Real wage	GDP	Unemp.	Real wage
GDP	1			1		
Unemp.	−0.78	1		−0.56	1	
Real wage	0.17	−0.04	1	0.52	−0.33	1

Table 4: Cross-correlations

signs are always correct, and the magnitudes are not very far off. Interestingly, the unemployment/real wage correlation is underpredicted for the Euro Area, while it is overpredicted for the USA. But the lack of correlation for these two variables in US data is puzzling, and means that the model has a hard time replicating this, using unconditional shocks processes. We pick up this issue in the discussion of the variance decomposition.

	Euro Area					
	Data			Model		
Time	GDP	Unempl.	Real wage	GDP	Unempl.	Real wage
1	0.75	0.93	0.8	0.62	0.95	0.79
2	0.5	0.81	0.65	0.4	0.86	0.55
3	0.33	0.67	0.47	0.27	0.76	0.36
4	0.2	0.52	0.28	0.2	0.66	0.23
5	0.03	0.35	0.17	0.15	0.56	0.14
	U.S.					
	Data			Model		
Time	GDP	Unempl.	Real wage	GDP	Unempl.	Real wage
1	0.84	0.87	0.83	0.65	0.88	0.83
2	0.64	0.7	0.65	0.44	0.76	0.6
3	0.41	0.51	0.51	0.3	0.67	0.41
4	0.23	0.33	0.41	0.22	0.58	0.27
5	0.06	0.16	0.27	0.17	0.51	0.17

Table 5: Autocorrelations

The model autocorrelations are broadly in line with the data, except for unemployment in the US: it is much more persistent in the model than in the data. The low-order autocorrelations are somewhat lower in the model than in the data for both regions, but the model captures persistance quite well. The real wage process is well matched in both regions.

The final criterion in evaluating the model is the theoretical variance decomposition for the observed variables. Here, as in Lubik (2009), the shortcoming of the baseline search-and-matching framework stands out. As Table 6 shows, unemployment volatility is exclusively driven by the matching function shock in both regions. This means that there is very little internal propagation in the model, neither for productivity shocks nor for price markup shocks. The model does better along this dimension for GDP and the real wage: while their are driven mostly by their "own" shocks, both productivity and markup shocks are propagated to the other variables. The matching function shock, however, plays no role in GDP or real wage volatility, confirming the disconnect between these two variables and unemployment.

It would be mistaken, though, to regard the job creation condition as playing no role in the dynamics of unemployment. While the matching function shock mechanically increases the number of matches for given vacancies and unemployment, there is also a response of vacancies to the falling marginal hiring costs. This is the same logic as in a model with investment adjustment costs, where there is a direct and an indirect effect of investment-specific productivity shocks. However, this effect is not likely to be very strong, since the model fails to generate a Beveridge curve, which should reflect a sustained high incentive to post vacancies even when unemployment is falling.

To summarize, our estimation results indicate that (i) real wage rigidity in a representative agent framework is not important to explain unemployment volatility in response to productivity or markup shocks, (ii) average real wages are not particularly rigid, although a bit more so in the EA than in the US, (iii) observed relative volatilities are explained by shocks other than a technology shock, (iv) the baseline model does not explain unemployment volatility, instead it leaves it as a residual.

	Euro Area		
	Productivity	Markup	Matching fn.
Output	82.24	0.18	17.58
Unemployment	0.44	1.01	98.55
Real wage	26.98	71.55	1.47
	U.S.		
	Productivity	Markup	Matching fn.
Output	77.92	0.86	21.21
Unemployment	1.14	3.87	94.99
Real wage	21.68	76.55	1.78

Table 6: Variance decomposition

3.4 Impulse responses

The impulse response analysis shows the differential impacts of shocks on output and various labor market variables. In Figure 1, we display the response of the U.S. and the Euro Area variables to technology shocks, at the estimated parameter values. While the shock process and output behave almost identically, one can see a striking difference between the two areas: in the U.S. unemployment and the aggregate wage respond much more strongly to technology shocks. This is partly the result of differential wage rigidities for incumbent workers, but largely the outcome of the different calibrations of the labor market variables, such as the unemployment rates and job finding rates. The U.S. labor market responds much more strongly to shocks.

The reason for the systematic difference can be clearly seen from the linearized job creation equation (13). For simplicity, we reproduce it here for the case with new hires wages perfectly flexible, $\vartheta_w = 0$, which is close to the estimated values:

$$\hat{q}_t = \beta (1 - \rho) E_t \hat{q}_{t+1} - \frac{\bar{q}}{\kappa} \left[\bar{x}\bar{z} (\hat{x}_t + z_t) - \bar{w}\hat{\omega}_t \right].$$

Recall that $\hat{q}_t = (\vartheta - 1)\hat{\theta}_t$ and $\hat{\theta}_t = \hat{v}_t - \hat{u}_{t-1}$, so that lower values of $\hat{q}_t$ are associated with higher values of $\hat{\theta}_t$ and $\hat{v}_t$. Furthermore, higher values of $\hat{\theta}_t$ mean that unemployment falls in subsequent periods. The key parameters governing the response of job creation – probability of filling a job – are $\bar{q}$ and κ. For the U.S., $\bar{q}/\kappa$ is 5.82, while for the Euro Area, we have that $\bar{q}/\kappa = 2.34$. This largely explains the much larger response of unemployment to productivity shocks.

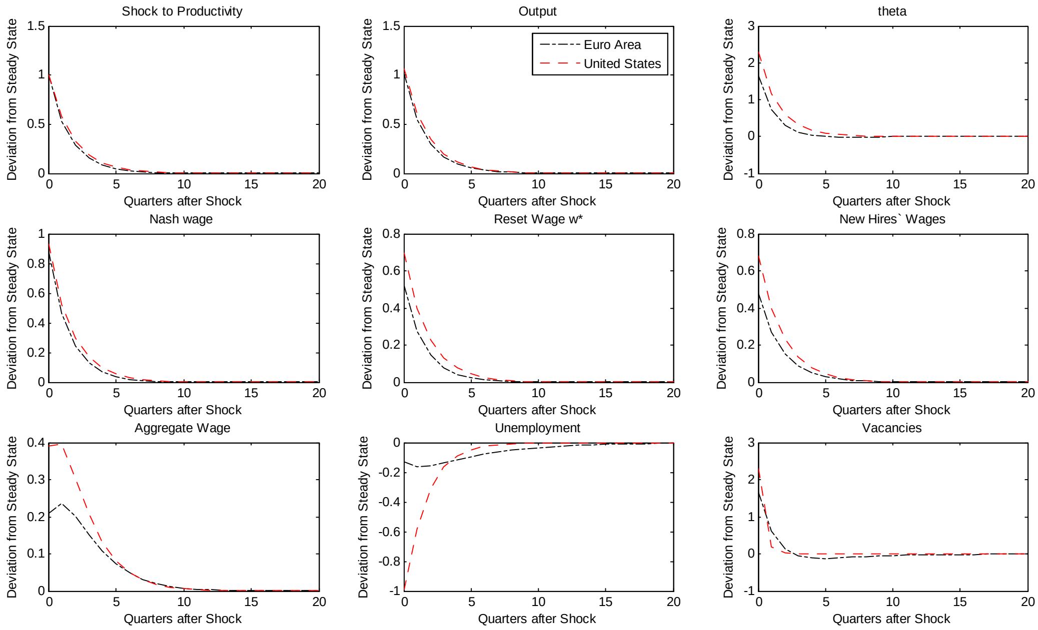

Figure 1: Impulse responses to productivity shocks in U.S. and in Euro Area

The stronger response of aggregate wages in the U.S. is due to two factors. First, the newly set wage w_t^* is more cyclical. Since wages for incumbent workers are more flexible in the U.S., they follow more closely the Nash wage ω_t, than in the Euro Area. Recall, that the newly set wage is a weighted average of all future Nash wages, with weights depending on the reset probabilities. The second factor is a composition effect. Since labor market turnover in the U.S. is higher than in the Euro Area, more new employment relationships are formed each period. In Europe only 1.4 percent of jobs are filled with unemployed workers, but in the U.S. 5.1 percent are.[9] This corresponds to the differences across countries in the separation rates ρ. Therefore, a larger fraction of the average wage is determined by the more cyclical wages of new hires, which move very closely with reset wages w_t^*.

[9] Of course, this ignores flows of workers from job to job. But also here, flows appear to be larger in the U.S. and the composition effect would make newly hired wages more cyclical.

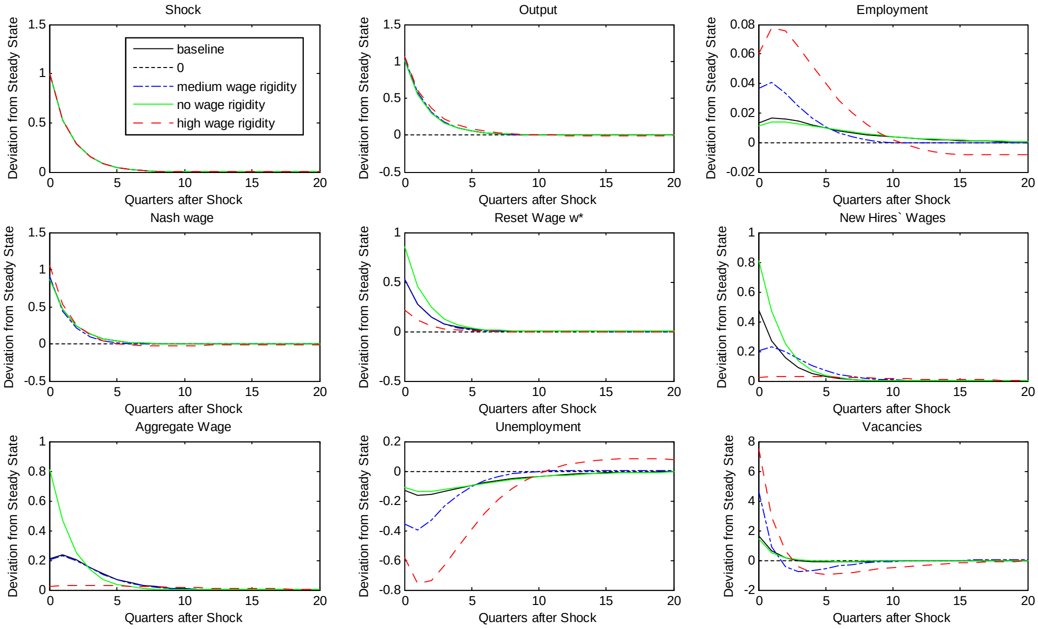

Figure 2: Impulse responses to technology shocks for different degrees of real wage rigidity: Euro Area

3.5 The role of real wage rigidity

In the model where wages are continuously renegotiated in response to changes in the economic environment, real wages respond directly to revenue and labor market tightness, as can be seen in equation (10). If there is wage rigidity for existing workers only, this close link breaks. Since newly negotiated wages are expected to be fixed for some time, they are set so as to be correct on average. (See equation 11). Thus, newly negotiated wages must be less volatile than the notional Nash wages. At the same time, of course, the present value of newly set wages must be identical to that of the frictionless, notional wages, because the present value of wage income is determined by the Nash bargaining equation (7).

This sections illustrates the role of the different degrees of real wage rigidity that we introduced in the previous sections. Again, the focus is on productivity shocks in the U.S. and the E.U. Figure 2 shows the impulse response in the Euro Area, for four different cases: the baseline case as estimated (black, straight line), a case with new hires' wages as rigid as the estimated rigidity of incumbent wages (blue, dash-dotted line), a case with virtually no rigidity (green, dotted line), and a case with extremely rigid wages for both types of employees, with γ_w and ϑ_w at 0.9 (red, dashed line). Figure 3 shows the corresponding responses in the U.S.

For the Euro Area, the first thing to note is that the baseline case with wage rigidity only for incumbent workers, and the case with no wage rigidity for incumbent workers are very similar. This shows the irrelevance of existing workers wages for employment dynamics. Only the dynamics of wages as such are influenced by the fact that existing workers wages are flexible. Thus the reset wage w^* reacts more strongly to shocks, and moves more closely with the Nash wage. In this case, also newly hired workers wages move more strongly, identical to the reset wage.

Secondly, we see that higher wage rigidity for new hires raises employment and unemployment volatility, as well as the volatility of vacancies. In contrast, wage volatility falls. New hires' wages are almost constant, and so are aggregate wages. Nevertheless, the volatility of output barely changes, and remains driven by the productivity shock alone. Furthermore, the impulse response suggests that even for high wage rigidity, unemployment does not even reach the volatility of productivity. The propagation of productivity shocks remains low. The reason for this is obviously not the rigidity of wages per se, but a labor market that is characterized by low job finding rates and low separation rates. Of course, various labor market institutions jointly determine labor market dynamics, but the results suggest that wage rigidity may not be the main determinant of sluggish labor market adjustment.

In constrast, the U.S. labor market is much more responsive to counterfactual changes in real wage rigidity. A very high degree of real wage rigidity significantly amplifies the volatility of unemployment and vacancies. As a larger fraction of employment relationships is newly generated each period, the stronger incentives to post vacancies when wages are rigid carry a much higher weight for unemployment dynamics. This leads to the strong response of vacancies. Thus there is amplification of output movements beyond the productivity shock. The high wage rigidity leads to a slight overshooting of the unemployment rate after the shock has faded. When the shock has faded, wages do not adjust fast enough back to the steady state level. Thus incentives to post vacancies are subdued for some time, leading to a lower $\theta_t = v_t/u_t$, and thus also lower Nash wages.

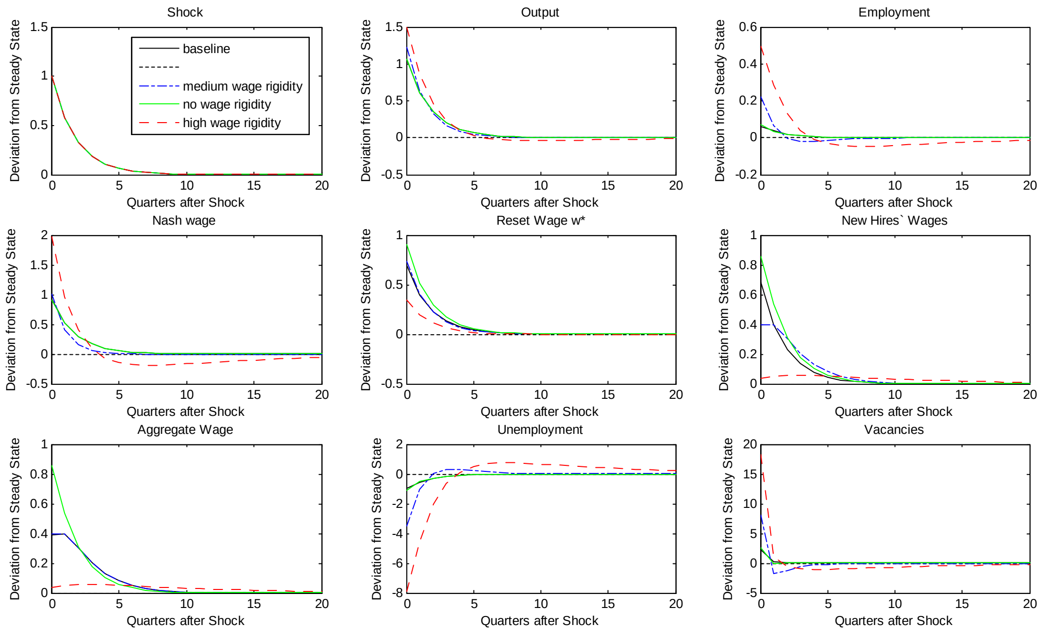

Figure 3: Impulse responses to technology shocks for different degrees of real wage rigidity: United States

3.6 Robustness

With the baseline findings at hand, we proceed to examine their robustness to changes in the estimation exercise, and the inclusion of an additional observable, vacancies. Vacancies are only available for the US, so our findings are suggestive at best for the Euro Area.

3.6.1 Variations in the baseline procedure

We reestimated the model with a number of modifications, to see how our baseline results change. Since our findings are remarkably similar for the Euro Area and the USA, we report only Euro Area results. Also, since our goal is testing the robustness of the baseline scenario, we do not present the alternative estimation results, we only highlight differences where they are significant. Numbers pertaining to all specifications are available from the authors upon request.

As a reminder, we list our baseline choices again: we calibrated $\vartheta = 0.5$ (matching function elasticity) and $b_u = 0.6$ (unemployment replacement rate), and estimated the wage Calvo parameters ϑ_w (new hires) and γ_w (old hires), along with η (the bargaining power of workers). Whenever we estimate a parameter, we keep the baseline priors, but in different specifications we alternate the calibrated and estimated parameters. Apart from the changes listed for each specification, we always keep the treatment of other parameters as in the baseline. We reestimate the model with the following changes:

Model 1 Calibrate the matching function elasticity to $\vartheta = 0.68$, which we get by regressing the job finding rate on labor market tightness using the US JOLTS dataset

Model 2 Calibrate the unemployment replacement rate $b_u = 0.4$, since this parameter is controversial in the literature, as it may reflect unemployment income or utility other than benefits

Model 3 Estimate both bargaining power η and b_u, instead of calibrating the latter

Model 4 Calibrate bargaining power $\eta = 0.5$ but estimate unemployment benefit b_u

Model 5 Restrict wage rigidities to be $\gamma_w = \vartheta_w = 0$, i.e., estimate the flexible wage version

Overall, the parameter estimates and our other conclusions are very robust to these modifications. The estimated wage rigidity for existing jobs (γ_w) is in the range of $0.55 - 0.65$, with the obvious exception of model 5. Wage rigidity for new hires (ϑ_w) is always below 0.1, except for model 4 where it is 0.25 (but with a 90% confidence interval of $0.03 - 0.46$).

Apparently there is some tradeoff between new hires wage rigidity and workers' bargaining power. Interestingly it is not the case for the replacement rate b_u. Thus our estimates do not indicate that the replacement rate is particularly high, and that high values are needed to match observed unemployment volatility. Given, however, that there is little propagation of technology shocks into unemployment in our estimated model, this conclusion is only valid for the unconditional volatility.

The joint estimation of η and b_u leaves the former essentially unchanged relative to the baseline, but the estimated replacement rate is only 0.46 $(0.3 - 0.62)$. Since the confidence interval (just) contains our baseline value, and since other aspects of the estimation and simulation are unchanged, we conclude that there is some uncertainty about the precise value of the replacement rate, but the model is not sensitive to it, at least in the range we explored.

Finally, we can reject the complete flexibility of the real wage for existing hires. The log data density for the baseline model is 1024.4, while in model 5 it is merely 996 – a fairly big difference. Also, whenever we estimate γ_w, the confidence intervals are always quite narrow, similarly to our baseline (Table 2). For models 1 to 4, the log data densities are much closer to the baseline, with model 2 being the lowest at 1017.2. None of the alternative models seem to be superior to our baseline specification: the only case when the log data density is higher is model 3 with 1025.2.

3.6.2 Using vacancy data (US)

Our search-and-matching model with exogenous separations assigns a crucial role to vacancy creation. Thus it is highly desirable to include vacancy data in the estimation. Also, an important empirical observation across countries is the negative correlation between unemployment and vacancies, the so-called Beveridge curve. Our baseline model is unable to reproduce this, in fact it predicts a positive correlation (not shown), due to the presence of shocks other than the technology shock. Using vacancy data should also help in the reproduction of the Beveridge curve.

Unfortunately, such data do not yet exist for the Euro Area, and it is too heterogenous and incompatible across European countries for aggregation. Thus we are forced to concentrate on the US in this section. Even in the US, high quality vacancy data is recent, provided by the JOLTS survey from the end of 2000. Since we want to keep our original sample, we use the help-wanted index historically used for vacancies. Fortunately, we do not need vacancy levels, only business cycle fluctuations, for which the earlier data is sufficient.

In order to estimate the model with 4 observables (GDP, unemployment, real wage, vacancies), we need to add a fourth shock to the model. We choose to include a shock to the outside option of workers, making b_t stochastic. This shock enters the equation of the Nash wage, and hence indirectly real wage change and job creation. It is akin to a labor supply shock in models with preferences that include labor. We assume the same AR(1) process as prior as we have used for the other shocks. All other aspects of the estimation procedure are unchanged, except that we estimate the average value $\bar{b}/\bar{w}$ (the replacement *rate*) along with γ_w, ϑ_w and η.

Parameter	**Prior mean**	**Posterior mean**	**90% Conf. interval**
U.S.			
γ_w	0.50	0.503	0.4281 – 0.5753
ϑ_w	0.50	0.0546	0.0068 – 0.1051
η	0.50	0.6031	0.4829 – 0.7290
$\bar{b}/\bar{w}$	0.50	0.4607	0.3294 – 0.5868
ρ_z	0.5	0.5972	0.4619 – 0.7571
ρ_{mf}	0.5	0.5398	0.4075 – 0.6846
ρ_μ	0.5	0.5001	0.3795 – 0.6137
ρ_b	0.5	0.8479	0.7884 – 0.9111
$s.d.\varepsilon^z$	0.05	0.0080	0.0078 – 0.0083
$s.d.\varepsilon^{mf}$	0.05	0.0265	0.0232 – 0.0291
$s.d.\varepsilon^\mu$	0.05	0.0173	0.0133 – 0.0215
$s.d.\varepsilon^b$	0.05	0.0943	0.0482 – 0.1445

Table 7: Estimation results with vacancies (US)

Table 7 presents the new estimates. The wage rigidity parameters are essentially unchanged, and the same is true for the technology and markup shock processes. The bargaining power of workers is lower than before, and the average replacement rate is reasonably close the the previously calibrated value. Interestingly, the previously high autocorrelation for the matching function shock is much lower now, and its role is taken up by the outside option shock.

We skip the detailed presentation of the data moments, which are in general reproduced by the model, as well as or even better than in our baseline specification. In particular, the standard deviation of unemployment is 0.082 in the model (0.086 in the data), and for vacancies it is 0.11 (0.11). The correlation between u and v is -0.75 in the model (-0.867 in the data), so now the Beveridge curve is also reproduced. The standard deviation of labor market tightness, $\theta = v - u$, is 0.155 in the model (0.19 in the data). Overall, we conclude that the model does a much better job now in reproducing vacancy creation.

	U.S.			
	Productivity	Markup	Matching fn.	Outside option
Output	80.79	1.14	4.93	13.14
Unemployment	1.5	5.83	25.29	67.37
Vacancies	2.83	12.65	10.40	74.12
Real wage	22.42	61.14	2.21	14.22

Table 8: Variance decomposition with vacancies (US)

The main improvement, however, can be seen from Table 8, which presents the variance decomposition. The matching function shock no longer dominates unemployment fluctuations, although it is still important. The main role in job creation, and hence also in unemployment, is played by the outside option shock. Output and real wage fluctuations are still explained mostly by their "own" shocks, but here also the outside option shock plays a non-negligible role.

One can see that employment movements are now generated to a much larger extent by economic motives rather than just the mechanical increase in matches due to matching shocks. For example, a fall in the outside option of workers reduces wages for new workers in particular and thus increase incentives to create jobs. With the expansion of employment comes a drop in unemployment, thus generating a Beveridge curve. Interestingly, even though it enters the real wage equation, the outside option shock does not play a significant role for real wage dynamics. As before, the shock to the markup is crucial to explain wage movements.

Overall, we conclude that the inclusion of a shock to workers' outside option and using vacancy data improves the model's fit significantly. The Beveridge curve is now implied by the model, and we also see improvements in the model's ability to match the main data moments. Apparently, the new shock captures an important aspect of the labor market, which drives job creation and also plays a role in output and real wage fluctuations. Lacking vacancy data, we cannot confirm the same for the Euro Area, but we see this as a promising direction in the evaluation of search-and-matching models. However, for our results on the rigidity of wages, the inclusion of vacancies is innoccous.

4 Conclusion

The degree of real wage rigidity has always been regarded as crucial for the understanding of aggregate employment fluctuations. With the observations of Hall (2005) and Shimer (2005) that the canonical search and matching model cannot explain the volatility of the labor market in response to productivity fluctuations, real wage rigidity has become a prime candidate to remedy this shortcoming. The key issue for the volatility of unemployment in the model is whether the wages of newly hired workers are in fact sufficiently rigid to amplify the propagation of shocks. We contribute to this question by estimating a business cycle model with search frictions and differential degrees of wage rigidity for newly hired and incumbent workers. We find that, using Euro Area and U.S. data, that Euro Area wages are somewhat more rigid than in the U.S., but that newly hired workers' wages are freely negotiated. Furthermore, in our model, the role of productivity shocks in explaining unemployment fluctuations is small in both regions.[10]

These findings imply that external labor markets appear to be as flexible in the Euro Area as they are in the United States, as far as new hires' wage rigidity is concerned. This is somewhat surprising, but does not imply that the labor markets are identical, because our models are calibrated to match differences in worker flows. On the other hand, wages for already employed workers are more rigid in the Euro Area than in the U.S. It should be noted that even though new workers' wages may be freely set initially, they are rigid for the same expected duration as for existing workers.

[10] Pissarides (2009) suggests instead a fixed component of recruitment costs which enables the model to generate plausible responses of both wages and unemployment to productivity shocks.

The shock decomposition points towards a limited role of transitory productivity movements for the cyclical dynamics of wages. Price markup shocks play a more important role, as found in other studies (Krause, Lopez-Salido, and Lubik, 2008).

Labor market dynamics in the model appear to be driven by shocks to the matching process and potentially worker outside options (as shown for the U.S. model with vacancies). This mirrors findings of Lubik (2009) and others who emphasize a separation between shocks that drive output movements and shocks that drive employment fluctuations.[11] The labor market appears to be driven by factors not captured well by productivity per se. This casts doubt on studies that focus on productivity shocks only, and statements that find a small response of the labor market to productivity shocks may be misleading.

In this paper, we have taken a purely aggregate perspective, as far as the data are concerned. It is somewhat surprising that, nevertheless, the model's parameters turn out so close to what some micro-studies appear to suggest. We find estimates that are in fact close to the extremes of total flexibility for new hires' wages, as found by Pissarides (2009) and Haefke, Sonntag and van Rhens (2008). Furthermore, it is encouraging that the estimations with the U.S. and Euro Area data in fact deliver parameter estimates in line with priors on the institutional differences in the U.S. and E.U. It remains to be seen whether detailed micro studies confirm this.

In contrast, our results differ from estimations of search and matching models with sticky prices, such as Gertler, Sala, and Trigari (2008). There, the estimated degree of wage rigidity is much higher. The difference is likely to be entirely due to the need to generate a higher degree of real wage and therefore real marginal cost rigidity to be able to explain inflation dynamics. Thus it may well be that real wage rigidity as such is not a feature of the labor market, but an artifact of the need to match other aspects of the data. It may also be an indication of a joint wage and price setting process that is not well captured by the conventional New Keynesian model with frictional labor markets.[12]

[11] Even though Lubik (2009) estimates all parameters of the model, including an elasticity of search costs, the results are remarkably similar, also for the variance decomposition.

[12] See Christoffel et al. (2009) for an assessment of alternative mechanisms.

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A Data

Euro Area

We use data from the Area Wide Model (Fagan, Henry and Mestre 2001), downloadable from http://www.eabcn.org/area-wide-model. We used the 2006 revision, which makes the time series available until 2005Q4. The following variables are used:

- Unemployment rate (URX)
- GDP (YER)
- GDP deflator (YED)
- Nominal wage rate (WRN)
- Labor force (LFN)

We compute the real wage by deflating WRN with YED. Real GDP per capita is calculated by delfating YER with YED, and dividing by LFN (which we convert to an index number). Finally, we apply the HP filter to logs of the variables using a smoothing parameter of 1,600 recommended for quarterly data.

United States

Our data comes chiefly from the Bureau of Labor Statistics and from the FRED database of the St Louis FED. For vacancies we use data from the OECD.Stat Extracts (http://stats.oecd.org/index.aspx). The following series are used:

- Hourly compensation, non-farm business sector (PRS85006103, BLS)
- Average weekly hourse, non-farm business sector (PRS85006023, BLS)
- GDP deflator (GDPDEF, FRED)
- Real GDP (GDPC96, FRED)
- Civilian non-institutional population >16 (LNS10000000, BLS)
- Unemployment rate (UNRATE, FRED)
- Vacancies, Conference Board help-wanted index (Job Vacancies, OECD)

The real wage is computed by deflating PRS85006103 with GDPDEF, and multiplying with PRS85006023. Real GDP per capita is divided by an index created from LNS10000000. As for the Euro Area, we apply the HP filter to the logs of variables with a smoothing parameter of 1,600.

B Labor market derivations

Firms

- Firm surplus for a newly set wage

$$
\begin{aligned}
J_t\left(w_t^*\right) &= \xi_t - w_t^* + \beta\left(1-\rho\right)E_t\left[\gamma_w J_{t+1}\left(w_t^*\right) + \left(1-\gamma_w\right)J_{t+1}\left(w_{t+1}^*\right)\right] \\
J_t\left(w_{t-1}^*\right) &= \xi_t - w_{t-1}^* + \beta\left(1-\rho\right)E_t\left[\gamma_w J_{t+1}\left(w_{t-1}^*\right) + \left(1-\gamma_w\right)J_{t+1}\left(w_{t+1}^*\right)\right] \\
J_t\left(w_t^*\right) - J_t\left(w_{t-1}^*\right) &= -\left(w_t^* - w_{t-1}^*\right) + \beta\gamma_w\left(1-\rho\right)E_t\left[J_{t+1}\left(w_t^*\right) - J_{t+1}\left(w_{t-1}^*\right)\right] \\
&= -\left(w_t^* - w_{t-1}^*\right)E_t\sum_{j=0}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j \\
J_t\left(w_t^*\right) &= \xi_t - w_t^* + \beta\left(1-\rho\right)E_t J_{t+1}\left(w_{t+1}^*\right) + E_t\left(w_{t+1}^* - w_t^*\right)\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j
\end{aligned}
$$

- New hires without newly set wages

$$
\begin{aligned}
J_t\left(w_{t-1}\right) &= \xi_t - w_{t-1} + \beta\left(1-\rho\right)E_t\left[\gamma_w J_{t+1}\left(w_{t-1}\right) + \left(1-\gamma_w\right)J_{t+1}\left(w_{t+1}^*\right)\right] \\
J_t\left(w_t^*\right) - J_t\left(w_{t-1}\right) &= -\left(w_t^* - w_{t-1}\right) + \beta\gamma_w\left(1-\rho\right)E_t\left[J_{t+1}\left(w_t^*\right) - J_{t+1}\left(w_{t-1}\right)\right] \\
&= -\left(w_t^* - w_{t-1}\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j \\
J_t\left(w_{t-1}\right) &= J_t\left(w_t^*\right) + \left(w_t^* - w_{t-1}\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j
\end{aligned}
$$

- Vacancy condition

$$
\begin{aligned}
V_t &= -\kappa + q_t\left[\vartheta_w J_t\left(w_{t-1}\right) + \left(1-\vartheta_w\right)J_t\left(w_t^*\right)\right] \\
&= -\kappa + q_t J_t\left(w_t^*\right) + q_t\vartheta_w\left(w_t^* - w_{t-1}\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j \\
\frac{\kappa}{q_t} &= J_t\left(w_t^*\right) + \vartheta_w\left(w_t^* - w_{t-1}\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j
\end{aligned}
$$

- Value of a job once more

$$\begin{aligned} J_t(w_t^*) &= \xi_t - w_t^* + E_t\left(w_{t+1}^* - w_t^*\right)\sum_{j=1}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j + \beta(1-\rho)E_tJ_{t+1}\left(w_{t+1}^*\right) \\ &= \xi_t - w_t^* + E_t\left(w_{t+1}^* - w_t^*\right)\sum_{j=1}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j \\ &\quad +\beta(1-\rho)E_t\left[\frac{\kappa}{q_{t+1}} - \vartheta_w\left(w_{t+1}^* - w_t\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j\right] \\ &= \xi_t - w_t^* + \beta(1-\rho)E_t\frac{\kappa}{q_{t+1}} \\ &\quad + \left[E_t\left(w_{t+1}^* - w_t^*\right) - \frac{\vartheta_w}{\gamma_w}\left(w_{t+1}^* - w_t\right)\right]\sum_{j=1}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j \end{aligned}$$

- Job creation

$$\begin{aligned} \frac{\kappa}{q_t} &= \xi_t - w_t^* + E_t\frac{\beta(1-\rho)\kappa}{q_{t+1}} + \left[E_t\left(w_{t+1}^* - w_t^*\right) - \frac{\vartheta_w}{\gamma_w}\left(w_{t+1}^* - w_t\right)\right]\sum_{j=1}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j \\ &\quad +\vartheta_w\left(w_t^* - w_{t-1}\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j \end{aligned}$$

Workers

- Newly set wages

$$\begin{aligned} W_t(w_t^*) &= w_t^* + \beta E_t\left\{(1-\rho)\left[\gamma_w W_{t+1}(w_t^*) + (1-\gamma_w)W_{t+1}\left(w_{t+1}^*\right)\right] + \rho U_{t+1}\right\} \\ W_t\left(w_{t-1}^*\right) &= w_{t-1}^* + \beta E_t\left\{(1-\rho)\left[\gamma_w W_{t+1}\left(w_{t-1}^*\right) + (1-\gamma_w)W_{t+1}\left(w_{t+1}^*\right)\right] + \rho U_{t+1}\right\} \\ W_t(w_t^*) - W_t\left(w_{t-1}^*\right) &= w_t^* - w_{t-1}^* + \beta(1-\rho)\gamma_w E_t\left[W_{t+1}(w_t^*) - W_{t+1}\left(w_{t-1}^*\right)\right] \\ &= \left(w_t^* - w_{t-1}^*\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j \\ W_t(w_t^*) &= w_t^* + \beta E_t\left[(1-\rho)W_{t+1}\left(w_{t+1}^*\right) + \rho U_{t+1}\right] \\ &\quad -E_t\left(w_{t+1}^* - w_t^*\right)\sum_{j=1}^{\infty}\left[\beta\gamma_w(1-\rho)\right]^j \end{aligned}$$

- New jobs but wages not reset (derivation same as for firms)

$$W_t(w_{t-1}) = W_t(w_t^*) - (w_t^* - w_{t-1}) \sum_{j=0}^{\infty} [\beta\gamma_w (1-\rho)]^j$$

- Unemployment and net gain

$$\begin{aligned}
U_t &= b_t + \beta E_t \left\{ s_{t+1} \left[\vartheta_w W_{t+1}(w_t) + (1-\vartheta_w) W_{t+1}\left(w_{t+1}^*\right) \right] + (1-s_{t+1}) U_{t+1} \right\} \\
&= b_t + \beta E_t \left[s_{t+1} W_{t+1}\left(w_{t+1}^*\right) + (1-s_t) U_{t+1} \right] \\
&\quad -\beta\vartheta_w E_t s_{t+1}\left(w_{t+1}^* - w_t\right) \sum_{j=0}^{\infty} [\beta\gamma_w (1-\rho)]^j \\
&= b_t + \beta E_t \left[s_{t+1} W_{t+1}\left(w_{t+1}^*\right) + (1-s_t) U_{t+1} \right] \\
&\quad -E_t \frac{s_{t+1}\vartheta_w}{(1-\rho)\gamma_w} \left(w_{t+1}^* - w_t\right) \sum_{j=1}^{\infty} [\beta\gamma_w (1-\rho)]^j \\
W_t(w_t^*) - U_t &= w_t^* - b_t + \beta E_t (1-\rho-s_{t+1}) \left[W_{t+1}\left(w_{t+1}^*\right) - U_{t+1} \right] \\
&\quad - \left[E_t \left(w_{t+1}^* - w_t^*\right) - E_t \frac{s_{t+1}\vartheta_w}{(1-\rho)\gamma_w} \left(w_{t+1}^* - w_t\right) \right] \sum_{j=1}^{\infty} [\beta\gamma_w (1-\rho)]^j
\end{aligned}$$

Wage setting

- Wage equation derivation

$$\max \left[W_t(w_t^*) - U_t \right]^{\eta} J_t(w_t^*)^{1-\eta}$$

$$\eta J_t(w_t^*) = (1-\eta) \left[W_t(w_t^*) - U_t \right]$$

$$\begin{aligned}
&\eta \left\{ \xi_t - w_t^* + \beta(1-\rho) E_t \frac{\kappa}{q_{t+1}} + \left[E_t \left(w_{t+1}^* - w_t^*\right) - \frac{\vartheta_w}{\gamma_w} \left(w_{t+1}^* - w_t\right) \right] \sum_{j=1}^{\infty} [\beta\gamma_w (1-\rho)]^j \right\} = \\
&= (1-\eta) \left\{ w_t^* - b_t - \left[E_t \left(w_{t+1}^* - w_t^*\right) - E_t \frac{s_{t+1}\vartheta_w}{(1-\rho)\gamma_w} \left(w_{t+1}^* - w_t\right) \right] \sum_{j=1}^{\infty} [\beta\gamma_w (1-\rho)]^j \right\} \\
&\qquad + \eta\beta E_t (1-\rho-s_{t+1}) J_{t+1}\left(w_{t+1}^*\right)
\end{aligned}$$

$$\eta\left\{\xi_t - w_t^* + \beta\kappa E_t\theta_{t+1} + \left[E_t\left(w_{t+1}^* - w_t^*\right) - \frac{\vartheta_w}{\gamma_w}\left(w_{t+1}^* - w_t\right)\right]\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j\right\} =$$

$$= (1-\eta)\left\{w_t^* - b_t - \left[E_t\left(w_{t+1}^* - w_t^*\right) - E_t\frac{s_{t+1}\vartheta_w}{(1-\rho)\gamma_w}\left(w_{t+1}^* - w_t\right)\right]\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j\right\}$$

$$-\eta\beta E_t\left(1-\rho-s_{t+1}\right)\vartheta_w\left(w_{t+1}^* - w_t\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j$$

$$\eta\left\{\xi_t - w_t^* + \beta\kappa E_t\theta_{t+1} + \left[E_t\left(w_{t+1}^* - w_t^*\right) - \frac{\vartheta_w}{\gamma_w}\left(w_{t+1}^* - w_t\right)\right]\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j\right\} =$$

$$= (1-\eta)\left\{w_t^* - b_t - \left[E_t\left(w_{t+1}^* - w_t^*\right) - E_t\frac{s_{t+1}\vartheta_w}{(1-\rho)\gamma_w}\left(w_{t+1}^* - w_t\right)\right]\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j\right\}$$

$$-\eta E_t\frac{\left(1-\rho-s_{t+1}\right)\vartheta_w}{(1-\rho)\gamma_w}\left(w_{t+1}^* - w_t\right)\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j$$

Wage equation

$$w_t^* = \eta\left(\xi_t + \beta\kappa E_t\theta_{t+1}\right) + (1-\eta)\,b_t + E_t\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j\left[w_{t+1}^* - w_t^* - \frac{s_{t+1}\vartheta_w}{(1-\rho)\gamma_w}\left(w_{t+1}^* - w_t\right)\right]$$

Job creation

- Recall from earlier

$$\frac{\kappa}{q_t} = \xi_t - w_t^* + E_t\frac{\beta(1-\rho)\kappa}{q_{t+1}} + \left[E_t\left(w_{t+1}^* - w_t^*\right) - \frac{\vartheta_w}{\gamma_w}\left(w_{t+1}^* - w_t\right)\right]\sum_{j=1}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j$$
$$+\vartheta_w\left(w_t^* - w_{t-1}\right)\sum_{j=0}^{\infty}\left[\beta\gamma_w\left(1-\rho\right)\right]^j$$

- Define the Nash wage as

$$\omega_t = \eta\left(\xi_t + \beta\kappa E_t\theta_{t+1}\right) + (1-\eta)\,b_t$$

- **Job creation condition**

$$\frac{\kappa}{q_t} = \xi_t - \omega_t + E_t \frac{\beta(1-\rho)\kappa}{q_{t+1}} - E_t \sum_{j=1}^{\infty} [\beta\gamma_w(1-\rho)]^j \frac{(1-s_{t+1})\vartheta_w}{(1-\rho)\gamma_w} \left(w_{t+1}^* - w_t\right)$$
$$+\vartheta_w (w_t^* - w_{t-1}) \sum_{j=0}^{\infty} [\beta\gamma_w(1-\rho)]^j$$

Log-linear equations

- Wage equation

$$\hat{w}_t^* = \hat{\omega}_t + \left[E_t\left(\hat{w}_{t+1}^* - \hat{w}_t^*\right) - \frac{\vartheta_w}{\gamma_w}\bar{s}E_t\left(\hat{w}_{t+1}^* - \hat{w}_t\right)\right]\frac{\beta\gamma_w(1-\rho)}{1-\beta\gamma_w(1-\rho)}$$

- Average wage

$$\hat{w}_t = [\rho\vartheta_w + (1-\rho)\gamma_w]\,\hat{w}_{t-1} + [1 - \rho\vartheta_w - (1-\rho)\gamma_w]\,\hat{w}_t^*$$

- Wage Phillips curve derivation:

$$\begin{aligned}
\frac{\hat{w}_t-[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{w}_{t-1}}{1-\rho\vartheta_w-(1-\rho)\gamma_w} &= \hat{\omega}_t+\frac{\beta\gamma_w(1-\rho)}{1-\beta\gamma_w(1-\rho)}\cdot \\
&\left\{\frac{E_t\hat{w}_{t+1}-[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{w}_t}{1-\rho\vartheta_w-(1-\rho)\gamma_w}\right. \\
&\left.-\frac{\hat{w}_t-[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{w}_{t-1}}{1-\rho\vartheta_w-(1-\rho)\gamma_w}\right\} \\
&-\frac{\beta\vartheta_w(1-\rho)\bar{s}}{1-\beta\gamma_w(1-\rho)}\cdot \\
&\left\{\frac{E_t\hat{w}_{t+1}-[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{w}_t}{1-\rho\vartheta_w-(1-\rho)\gamma_w}-\hat{w}_t\right\} \\
\frac{[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{\pi}_t^w}{1-\rho\vartheta_w-(1-\rho)\gamma_w} &= (\hat{\omega}_t-\hat{w}_t) \\
&+\frac{\beta\gamma_w(1-\rho)}{1-\beta\gamma_w(1-\rho)}\frac{E_t\hat{\pi}_{t+1}^w-[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{\pi}_t^w}{1-\rho\vartheta_w-(1-\rho)\gamma_w} \\
&-\frac{\beta\vartheta_w(1-\rho)\bar{s}}{1-\beta\gamma_w(1-\rho)}\frac{E_t\hat{\pi}_{t+1}^w}{1-\rho\vartheta_w-(1-\rho)\gamma_w} \\
[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{\pi}_t^w &= [1-\rho\vartheta_w-(1-\rho)\gamma_w](\hat{\omega}_t-\hat{w}_t) \\
&+\frac{\beta\gamma_w(1-\rho)}{1-\beta\gamma_w(1-\rho)}\left[E_t\hat{\pi}_{t+1}^w-[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{\pi}_t^w\right] \\
&-\frac{\beta\vartheta_w(1-\rho)\bar{s}}{1-\beta\gamma_w(1-\rho)}E_t\hat{\pi}_{t+1}^w \\
[\rho\vartheta_w+(1-\rho)\gamma_w]\hat{\pi}_t^w &= [1-\rho\vartheta_w-(1-\rho)\gamma_w][1-\beta\gamma_w(1-\rho)](\hat{\omega}_t-\hat{w}_t) \\
&+\beta(1-\rho)(\gamma_w-\bar{s}\vartheta_w)E_t\hat{\pi}_{t+1}^w
\end{aligned}$$

- **Wage Phillips curve**

$$\hat{\pi}_t^w=\frac{\beta(1-\rho)(\gamma_w-\bar{s}\vartheta_w)}{\rho\vartheta_w+(1-\rho)\gamma_w}E_t\hat{\pi}_{t+1}^w+\frac{[1-\rho\vartheta_w-(1-\rho)\gamma_w][1-\beta\gamma_w(1-\rho)]}{\rho\vartheta_w+(1-\rho)\gamma_w}(\hat{\omega}_t-\hat{w}_t)$$

- Job creation condition derivation:

$$\begin{aligned}
w_t &= [\vartheta_w\nu_t+\gamma_w(1-\nu_t)]w_{t-1}+[1-\vartheta_w\nu_t-\gamma_w(1-\nu_t)]w_t^* \\
w_t^*-w_{t-1} &= \frac{w_t-w_{t-1}}{1-\vartheta_w\nu_t-\gamma_w(1-\nu_t)}
\end{aligned}$$

$$
\begin{aligned}
\frac{\kappa}{q_t} &= \beta E_t \lambda_{t+1} \Bigg\{ \xi_{t+1} - \omega_{t+1} + \frac{\kappa}{\lambda_{t+1} q_{t+1}} \\
&\quad - \frac{\vartheta_w}{\gamma_w} (1 - s_t) \left(w^*_{t+2} - w_{t+1} \right) \sum_{j=1}^{\infty} \frac{\lambda_{t+j+1}}{\lambda_{t+1}} \left[\beta \gamma_w (1-\rho) \right]^j \Bigg\} \\
&\quad + \frac{\vartheta_w}{\gamma_w} E_t \left(w^*_{t+1} - w_t \right) \sum_{j=1}^{\infty} \frac{\lambda_{t+j}}{\lambda_t} \left[\beta \gamma_w (1-\rho) \right]^j
\end{aligned}
$$

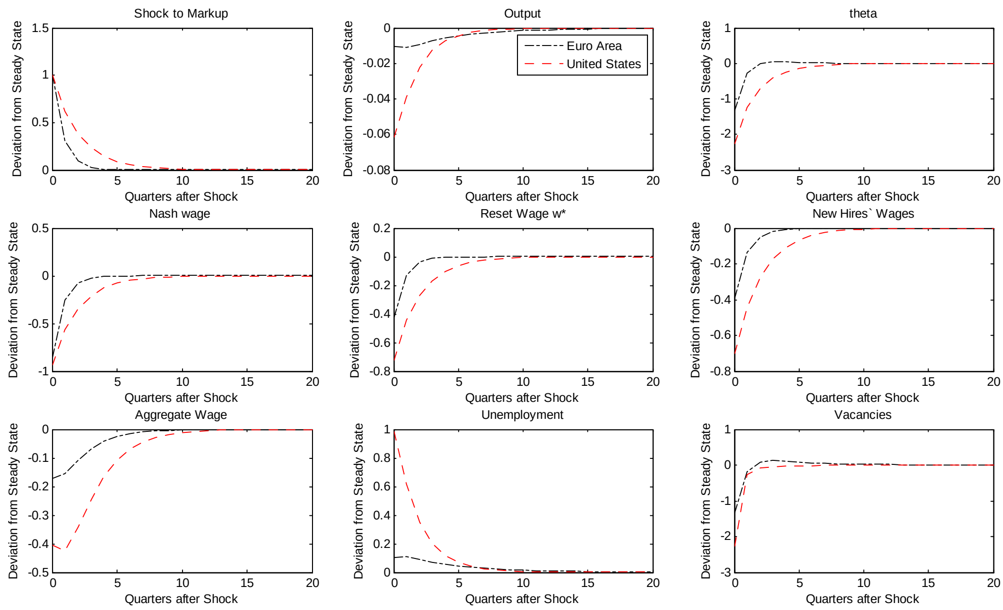
$$
\begin{aligned}
-\frac{\kappa}{\bar{q}} \hat{q}_t &= -\frac{\beta\kappa}{\bar{q}} E_t \hat{q}_{t+1} + \frac{(1-\beta)\kappa}{\bar{q}} E_t \hat{\lambda}_{t+1} + \beta \bar{\lambda} \left(\bar{\xi} E_t \hat{\xi}_{t+1} - \bar{w} E_t \hat{\omega}_{t+1} \right) \\
&\quad - \beta \bar{\lambda} \frac{\vartheta_w}{\gamma_w} (1 - \bar{s}) \frac{1}{1 - \vartheta_w \rho - \gamma_w (1-\rho)} \frac{\beta \gamma_w (1-\rho)}{1 - \beta \gamma_w (1-\rho)} E_t \bar{w} \hat{\pi}^w_{t+2} \\
&\quad + \frac{\vartheta_w}{\gamma_w} \frac{1}{1 - \vartheta_w \rho - \gamma_w (1-\rho)} \frac{\beta \gamma_w (1-\rho)}{1 - \beta \gamma_w (1-\rho)} E_t \bar{w} \hat{\pi}^w_{t+1}
\end{aligned}
$$

- **Job creation**

$$
\begin{aligned}
\hat{q}_t &= \beta E_t \hat{q}_{t+1} - (1-\beta) E_t \hat{\lambda}_{t+1} - \frac{\beta \bar{\lambda} \bar{q}}{\kappa} \left(\bar{\xi} E_t \hat{\xi}_{t+1} - \bar{w} E_t \hat{\omega}_{t+1} \right) \\
&\quad + \frac{\beta^2 \bar{\lambda} \bar{q} \vartheta_w (1-\rho)(1-\bar{s})}{\kappa \left[1 - \beta \gamma_w (1-\rho) \right] \left[1 - \vartheta_w \rho - \gamma_w (1-\rho) \right]} E_t \bar{w} \hat{\pi}^w_{t+2} \\
&\quad - \frac{\beta \bar{q} \vartheta_w (1-\rho)}{\kappa \left[1 - \beta \gamma_w (1-\rho) \right] \left[1 - \vartheta_w \rho - \gamma_w (1-\rho) \right]} E_t \bar{w} \hat{\pi}^w_{t+1}
\end{aligned}
$$

C Impulse responses

- Impulse response to markup shock



- Impulse response to matching efficiency